CRONUS-Earth ^{26}Al-^{10}Be exposure age calculator MATLAB function reference Version 2: November, 2007

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Accompanies Balco, G., and others, A straightforward, internally consistent, and easily accessible means of calculating surface exposure ages or erosion rates from ^{10}Be and ^{26}Al measurements.

November 13, 2007

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1.1 al_be_age_many_v2.m

Syntax:

```
retstr = al_be_age_many_v2(ins,localFlag)
```

The MATLAB web server calls this function when the exposure age data input form is submitted. It takes as input a structure containing string variables, which is supplied by the MATLAB web server. It returns a text string consisting of an output HTML document containing the results of the exposure age calculation. The documentation for the MATLAB web server describes this process in more detail.

There is only one required argument, the input structure `ins`. The fields are as follows:

ins.text_block	A text block containing sample information that follows the rules below.
ins.requesting_IP	The IP address of the requesting page. This is only required when localFlag is not set to 1, that is, when the function is in use on the web server.

The formatting rules for entering a text block containing data for multiple samples are as follows:

1. Enter plain ASCII text only.
2. Each sample should occupy its own line.
3. Each line should have thirteen elements, as described below.
4. Elements should be separated from each other by white space (spaces or tabs).
5. Something other than white space must be entered for each element. For example, if you have no ^{26}Al measurements for a sample, you must enter '0' in the ^{26}Al concentration and ^{26}Al uncertainty positions.

In most cases, pasting directly from an Excel spreadsheet should satisfy the rules. An example of an acceptable input text block appears below.

The thirteen elements are as follows. These are the same as the input parameters on the single-sample form.

1. Sample name. Any text string not exceeding 24 characters. Sample names may not contain white space or any characters that could be interpreted as delimiters or escape characters, e.g., slashes of both directions, commas, quotes, colons, etc. Stick to letters, numbers, and dashes.
2. Latitude. Decimal degrees.
3. Longitude. Decimal degrees.
4. Elevation/pressure. Meters or hPa, respectively, depending on selection below.

5. Elevation/pressure flag. Specifies how to treat the elevation/pressure value. This is a three-letter text string. If you have supplied elevations in meters and the default atmosphere is applicable at your site (locations outside Antarctica), enter 'std' here. If you have supplied elevations in meters and your site is in Antarctica, enter 'ant' here. If you have entered pressure in hPa, enter 'pre' here. Any text other than these three options will be rejected. For further discussion of altitude-atmospheric pressure relationships, see the main text of the paper.
6. Sample thickness. Centimeters.
7. Sample density. $g \cdot cm^{-3}$.
8. Shielding correction. Samples with no topographic shielding, enter 1. For shielded sites, enter a number between 0 and 1. The shielding correction can be calculated using `skyline.m`.
9. Erosion rate inferred from independent evidence. $cm \cdot yr^{-1}$.
10. ^{10}Be concentration. Atoms $\cdot g^{-1}$. Standard or scientific notation.
11. Uncertainty in ^{10}Be concentration. Atoms $\cdot g^{-1}$. Standard or scientific notation.
12. ^{26}Al concentration. Atoms $\cdot g^{-1}$. Standard or scientific notation.
13. Uncertainty in ^{26}Al concentration. Atoms $\cdot g^{-1}$. Standard or scientific notation.

Here is an example of an acceptable input text block:

```
PH-1 41.3567 -70.7348 91 std 4.5 2.7 1 8e-5 123500 3700 712400 31200
01-MBL-059-BBD -77.073 -145.686 712 ant 4.75 2.65 0.997 0 0 0 1.9e6 4.9e4
NH-1 57.968 -6.812 790 std 3 2.65 1 0 943000 28000 0 0
```

The optional argument `localFlag` turns off most of the server-specific system calls in this function. This option exists only for debugging the function externally to the web server, and is not very well generalized. If `localFlag` is set to 1, the function returns a structure of string variables containing only the chunks of HTML that report results, not a single string variable containing the the entire output web page. In addition, plots are created locally, and all of them have names beginning with 'temp.' Of course, not only does this result in repeated overwriting of plots, it requires GMT and ImageMagick to be running locally. Thus, calls to the plotting functions will generate numerous errors on many systems. This aspect of the code would need to be rewritten for general use.

The majority of this function consists of routines that check the input data to make sure it is in the expected form and is within expected bounds, and convert string variables to numerical values. After the data checking is complete, the function then assembles the data sets needed for the ^{26}Al and ^{10}Be exposure age calculations, loads the data file containing values for physical constants, and passes data to the function `get_al_be_age.m`, which actually carries out the exposure age calculation and returns the result. Finally, this function assembles the output data, calls additional functions to generate the required plots, inserts the output data into the output HTML template, and returns the web page containing the results.

The only actual calculation that takes place inside this function is the calculation of the uncertainty in the ^{26}Al ^{10}Be ratio. Denote the ^{26}Al/^{10}Be ratio by $R_{26/10}$ and its 1σ uncertainty by $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$(\sigma R_{26/10})^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2 \quad (1)$$

where N_i is the concentration of nuclide i and σN_i is its 1σ analytical uncertainty.

1.2 al_be_erosion_many_v2.m

Syntax:

```
retstr = al_be_erosion_many_v2(ins)
```

The MATLAB web server calls this function when the erosion rate data input form is submitted. It takes as input a structure containing string variables, which is supplied by the MATLAB web server. It returns a text string consisting of an output HTML document containing the results of the exposure age calculation. The documentation for the MATLAB web server describes this process in more detail.

There is only one required argument, the input structure `ins`. The fields are as follows:

ins.text_block	A text block containing sample information that follows the rules below.
ins.requesting_IP	The IP address of the requesting page. This is only required when localFlag is not set to 1, that is, if the function is in use on the web server.

The formatting rules for entering a text block containing data for multiple samples are as follows:

1. Enter plain ASCII text only.
2. Each sample should occupy its own line.
3. Each line should have twelve elements, as described below.
4. Elements should be separated from each other by white space (spaces or tabs).
5. Something other than white space must be entered for each element. For example, if you have no ^{26}Al measurements for a sample, you must enter '0' in the ^{26}Al concentration and ^{26}Al uncertainty positions.

In most cases, pasting directly from an Excel spreadsheet should satisfy the rules. An example of an acceptable input text block appears below.

The thirteen elements are as follows. These are the same as the input parameters on the single-sample form.

1. Sample name. Any text string not exceeding 24 characters. Sample names may not contain white space or any characters that could be interpreted as delimiters or escape characters, e.g., slashes of both directions, commas, quotes, colons, etc. Stick to letters, numbers, and dashes.
2. Latitude. Decimal degrees.
3. Longitude. Decimal degrees.
4. Elevation/pressure. Meters or hPa, respectively, depending on selection below.
5. Elevation/pressure flag. Specifies how to treat the elevation/pressure value. This is a three-letter text string. If you have supplied elevations in meters and the default atmosphere is applicable at your site (locations outside Antarctica), enter 'std' here. If you have supplied elevations in meters and your site is in Antarctica, enter 'ant' here. If you have entered pressure in hPa, enter 'pre' here. Any text other than these three options will be rejected. For additional discussion of elevation-atmospheric pressure relationships, see the text of the main paper.

6. Sample thickness. Centimeters.
7. Sample density. $g \cdot cm^{-3}$.
8. Shielding correction. Samples with no topographic shielding, enter 1. For shielded sites, enter a number between 0 and 1. The shielding correction can be calculated using `skyline.m`.
9. ^{10}Be concentration. Atoms $\cdot\ g^{-1}$. Standard or scientific notation.
10. Uncertainty in ^{10}Be concentration. Atoms $\cdot\ g^{-1}$. Standard or scientific notation.
11. ^{26}Al concentration. Atoms $\cdot\ g^{-1}$. Standard or scientific notation.
12. Uncertainty in ^{26}Al concentration. Atoms $\cdot\ g^{-1}$. Standard or scientific notation.

Here is an example of an acceptable input text block:

```
FV-TOP-1 38.6139 -109.1878 2527 std 2 2.5 1 4.59e5 1.3e4 2.69e6 8.80e4
FV-TOP-2 38.6136 -109.1955 2580 std 2 2.5 1 202000 8000 1100000 49000
FV-TOP-3 38.6204 -109.2062 2592 std 2 2.5 1 1.02e6 2.60e4 6.05e6 1.31e5
04-AV-PIT9-NEW -77.8282 160.9762 1300 ant 2 1.9 0.973 6710000 116000 0 0
```

The optional argument `localFlag` turns off most of the server-specific system calls in this function. This option exists only for debugging the function externally to the web server, and is not very well generalized. If `localFlag` is set to 1, the function returns a structure of string variables containing only the chunks of HTML that report results, not a single string variable containing the the entire output web page. In addition, plots are created locally, and all of them have names beginning with 'temp.' Of course, not only does this result in repeated overwriting of plots, it requires GMT and ImageMagick to be running locally. Thus, calls to the plotting functions will generate numerous errors on many systems. This aspect of the code would need to be rewritten for general use.

The majority of this function consists of routines that check the input data to make sure it is in the expected form and is within expected bounds, and convert string variables to numerical values. After the data checking is complete, the function then assembles the data sets needed for the ^{26}Al and ^{10}Be exposure age calculations, loads the data file containing values for physical constants, and passes data to the function `get_al_be_erosion.m`, which actually carries out the erosion rate calculation and returns the result. Finally, this function assembles the output data, generates the required plots, inserts the output data into the output HTML template, and returns the completed results web page.

The only actual calculation that takes place inside this function is the calculation of the uncertainty in the ^{26}Al ^{10}Be ratio. Denote the ^{26}Al/^{10}Be ratio by $R_{26/10}$ and its 1σ uncertainty by $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$(\sigma R_{26/10})^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2 \tag{2}$$

where N_i is the concentration of nuclide i and σN_i is its 1σ analytical uncertainty.

1.3 get_al_be_age.m

```
results = get_al_be_age(sample,consts,nuclide)
```

This is the main control function that carries out the exposure age calculation. `al_be_age_many_v2` calls it.

The argument `sample` is a structure containing sample information. The fields are as follows:

sample.sample_name	Sample name	string
sample.lat	Latitude	double
sample.long	Longitude	double
sample.elv	elevation in meters	double
sample.pressure	pressure in hPa (optional if sample.elv is set)	double
sample.aa	Flag that indicates how to interpret the elevation value.	string
sample.thick	Sample thickness in cm	double
sample.rho	Sample density, $g \cdot cm^{-3}$	double
sample.othercorr	Shielding correction.	double
sample.E	Independently measured erosion rate, $cm \cdot yr^{-1}$	double
sample.N10	^{10}Be concentration, atoms $\cdot g^{-1} \cdot yr^{-1}$	double
sample.delN10	standard error of ^{10}Be concentration	double
sample.N26	^{26}Al concentration, atoms $\cdot g^{-1} \cdot yr^{-1}$	double
sample.delN26	standard error of ^{26}Al concentration	double

The argument `consts` is a structure containing the constants. It is typically the structure created by `make_al_be_consts_v2.m`. See the documentation for `make_al_be_consts_v2.m` for a field list.

The argument `nuclide` tells the function which nuclide is being used; allowed values are 10 or 26. This argument is a numerical value, not a string.

This function returns a structure called `results` that contains the following fields:

Non-scaling-scheme-specific information:

results.flags	Non-fatal error messages, mostly to do with nuclide concentrations above saturation	string
results.main_version	Version number for this function	string
results.muon_version	Version of P_mu_total called internally	string
results.P_mu	Surface nuclide production rate due to muons (atoms · g^{-1}· yr^{-1})	double
results.thick_sf	Thickness scaling factor (nondimensional)	double
results.tv	Vector of time values (yr) used for plotting $P(t)$ and $Rc(t)$	1 x n double

Results pertaining to non-time-dependent (Lal 1991/Stone 2000) scaling scheme only:

results.SF_St_nominal	Pressure/latitude scaling factor according to Stone (2000). Mainly for historical interest.	double

Results pertaining to all scaling schemes:

results.t_XX	Five scalar values containing exposure ages (yr) calculated with the five scaling schemes. XX is replaced with 'St,' 'De,' 'Du,' 'Li,' or 'Lm' for the Lal(1991)/Stone(2000), Desilets (2006), Dunai (2000), Lifton (2005) and time-dependent Lal (1991)/Stone(2000) schemes.	double
results.delt_int_XX	Five scalar values containing internal uncertainties (yr) calculated with the five scaling schemes. Letter codes for each scheme as above. Note that these values should all be essentially the same.	double
results.delt_ext_XX	Five scalar values containing external uncertainties (yr) calculated with the five scaling schemes. Letter codes for each scheme as above.	double
results.Rc_XX	Four vectors (the same length as results.tv) containing cutoff rigidity values (Gv) calculated according to the four time-dependent scaling schemes. Letter codes for each scheme as above. Used for plotting $Rc(t)$.	1 x n double
results.P_XX	For the four time-dependent scaling schemes, these are vectors (the same length as results.tv) containing thickness-integrated surface production rates due to spallation (atoms · g^{-1}· yr^{-1}) according to the respective scaling schemes. Letter codes for each scheme as above. For the non-time-dependent St scaling scheme, this is a scalar value. Used for plotting $P(t)$.	1 x 1 or 1 x n double
results.FSF_XX	Five scalar values containing effective production rate scaling factors determined for the four time-dependent scaling schemes. The effective scaling factor is the scaling factor which, when inserted in the simple age equation, yields the age obtained from the full time-dependent calculation. The thickness and geometric scaling factors are not included; production by muons is included implicitly. Used in the error propagation scheme; reportd mainly for diagnostic purposes.	double

The exposure age calculation goes as follows:

1. Calculate a first estimate of the exposure age using the simple age equation.

Calculate the thickness scaling factor S_{thick} by calling the function `thickness.m`.

If `sample.pressure` is not set, calculate it by calling `NCEPatm_2.m` or `antatm.m`.

Calculate the geographic scaling factor according to Lal(1991)/Stone(2000) S_{St} by calling the function `stone2000.m`.

Obtain the surface production rate due to muons P_μ by calling `P_mu_total.m`.

For the non-time-dependent St scaling scheme, the production rate due to spallation in the sample $P_{sp,St}$ (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}) is :

$$P_{sp,St} = P_{ref,St} * S_{thick} * S_G * S_{St} \tag{3}$$

where $P_{ref,St}$ is the reference production rate for nuclide i given the Lal (1991)/Stone(2000) scaling scheme, and S_G is the geometric shielding correction. The surface production rate according to the St scaling scheme P_{St} is then $P_{sp,St} + P_\mu$.

Calculate the simple exposure age t_{simple}:

$$t_{simple} = \frac{1}{\lambda + \frac{\rho\epsilon}{\Lambda_{sp}}} \ln\left[1 - \frac{N}{P_{sp.St}}\left(\lambda + \frac{\rho\epsilon}{\Lambda_{sp}}\right)\right] \tag{4}$$

where N is the measured nuclide concentration (atoms $\cdot$ g^{-1}), ϵ is the erosion rate (cm $\cdot$ yr^{-1}), λ is the decay constant for the relevant nuclide (yr^{-1}), ρ is the sample density (g $\cdot$ cm^{-3}), and Λ_{sp} is the effective attenuation length for prodution by neutron spallation (g $\cdot$ cm^{-2}). The simple age does not reflect the different depth dependences of production by spallation and muons. This is taken into account later in the full forward calculation. Mainly the simple age is used only to estimate the length of the time domain for the forward calculation.

2. Calculate $P_{sp}(t)$ for the various scaling schemes.

In the following discussion, the various scaling schemes are denoted as ‘St’ for the non-time-dependent scheme of Lal (1991) and Stone (2000); ‘De’ for Desilets and others (2006); ‘Du’ for Dunai (2001); ‘Li’ for Lifton and others (2005); and ‘Lm’ for the time-dependent adaptation of the Lal (1991)/Stone(2000) scheme.

2.1. Create the time domain.

The length of the time domain for the forward calculation is $1.6 \cdot t_{simple}$. The factor of 1.6 is chosen to ensure that the time domain is always long enough, while not wasting effort calculating production rates for times much older than the exposure age of the sample.

The time steps for the forward age calculation follow the paleomagnetic data:

`[0:500:6500 6900]` to match the derivatives of the Korte and Constable field model. The nominal pole positions used in the Lm scaling scheme have been resampled to these times, as has the solar modulation factor used in the Li scaling scheme.

`[7500:1000:11500]` to match the paleointensities given in Yang et al. 2000. Again, solar modulation values up to 10,500 yr BP have again been resampled at these times.

`[12000:1000:800000]` to match the SINT800 paleointensity record.

`logspace(log10(810000),7,200)` gives 200 log-spaced points from 0.8 to 10 Myr. Note that 99% saturation occurs at 9.97 Myr for ^{10}Be and 4.69 Myr for ^{26}Al. This means that the calculation is not strictly accurate for near-saturated surfaces. Thus, this function is conservative about deciding whether or not an age can be calculated: measurements that are very close to saturation will probably be reported as saturated. If you are interested in a very precise evaluation of whether or not a particular measurement is saturated with respect to a particular scaling scheme, this code is not for you.

2.2. Calculate cutoff rigidity Rc as a function of time

We use the same magnetic field reconstruction for all the time-dependent scaling schemes; however, the different schemes use different methods for deriving the cutoff rigidity from the magnetic field parameters. From 0-7000 yr BP, the magnetic field is taken to be the spherical harmonic reconstruction of Korte and Constable. For 7000 - 800,000 BP, the field is taken to be a geocentric axial dipole with intensity given by Yang at al. (7000 - 11,500 BP) and SINT800 (11,500 - 800,000 BP). Prior to 800,000 BP, the magnetic field is assumed to be a geocentric axial dipole with the average intensity of the SINT800 record.

The main text of the paper gives additional details about the source of the magnetic field data and the derivatives thereof.

For the De scaling scheme, for 0-7000 yr BP, cutoff rigidity is directly interpolated for the sample site from the global grids of cutoff rigidity values obtained by trajectory tracing at 500-year intervals (Lifton and others (in prep) describe this calculation in more detail) Before 7000 BP, the cutoff rigidity R_C is calculated from the geographic latitude of the sample according to equation (19) of Desilets et al. (2003):

$$R_C = \sum_{i=0}^{6} \left[e_i + f_i \left(\frac{M}{M_0} \right) \right] \theta^{(i)} \qquad (5)$$

where (M/M_0) is the ratio of the field intensity at the past time of interest to the present field intensity and θ is the geographic latitude of the sample. The constants e_i and f_i are as follows:

i	e_i	f_i
0	$-4.3077x10^{-3}$	$1.4792x10^{+1}$
1	$2.4352x10^{-2}$	$-6.6799x10^{-2}$
2	$-4.6757x10^{-3}$	$3.5714x10^{-3}$
3	$3.3287x10^{-4}$	$2.8005x10^{-5}$
4	$-1.0993x10^{-5}$	$-2.3902x10^{-5}$
5	$1.7037x10^{-7}$	$6.6179x10^{-7}$
6	$-1.0043x10^{-9}$	$-5.2083x10^{-9}$

For the Du scaling scheme, for 0-7000 yr BP, cutoff rigidity is directly interpolated for the sample site from global grids of cutoff rigidity values that were obtained by applying equation (2) of Dunai (2001) to the horizontal field intensity and inclination of the Korte and Constable field model, again at 500-year intervals. We thank Nat Lifton for generating these grids. Before 7000 BP, the cutoff rigidity R_C is calculated from the geographic latitude of the sample according to equation (1) of Dunai (2001):

$$R_C = 14.9 \left(\frac{M}{M_0} \right) [cos(\theta)]^4 \qquad (6)$$

For the Li scaling scheme, for 0-7000 yr BP, cutoff rigidity is directly interpolated for the sample site from the global grids of cutoff rigidity values obtained by trajectory tracing. Before 7000 BP, the cutoff rigidity R_C is calculated from the geographic latitude of the sample according to equation (6) of Lifton and others (2005):

$$R_C = 15.765 \left(\frac{M}{M_0}\right) [cos(\theta)]^{3.8} \tag{7}$$

For the Lm scaling scheme, for 0-7000 yr BP, the geomagnetic latitude of the sample site is determined by approximating the Korte and Constable field model by a geocentric dipole. Again, we thank Nat Lifton for collating this information.The cutoff rigidity is then determined via equation (6) using the geomagnetic latitude for 0-7000 BP and the geographic latitude for earlier times.

2.3. Calculate the surface production rate as a function of time $P(t)$

Having obtained the cutoff rigidity as a function of time $R_C(t)$ appropriate to the various scaling schemes, we now obtain $P_{sp,Xx}(t)$ by calling the functions `desilets2006sp.m`, `dunai2001sp.m, \verb`lifton2006sp.m=, and `stone2000Rcsp.m` for the De, Du, Li, and Lm scaling schemes respectively, and multiplying the resulting vectors of scaling factors by the appropriate reference production rates. For the non-time-dependent St scaling scheme, $P_{sp,St}(t) = P_{ref,St} * S_{St}$, as calculated above, for all t.

3. Integrate forward; interpolate to obtain the exposure ages

Now that we have $P_{sp,Xx}(t)$ for the various scaling schemes (for 'Xx,' read one of the two-letter scaling scheme codes), we can calculate $N_{Xx}(t)$ by:

$$N_{Xx}(T) = S_{thick}S_G \int_0^T P_{sp,Xx}(t) \exp{(-\lambda t)} \exp\left(\frac{-\epsilon t}{\Lambda_{sp}}\right) dt + P_\mu \int_0^T \exp{(-\lambda t)} \exp\left(\frac{-\epsilon t}{\Lambda_\mu}\right) dt \tag{8}$$

Where we take the effective attenuation length for production by muons Λ_μ to be 1500 g $\cdot$ cm^{-2}. This simplification of the depth dependence of production by muons is acceptable because sites that can be accurately exposure dated are by definition sites with low erosion rates. In the erosion rate calculation, we use a more physically correct depth dependence for production by muons. Note also that the thickness scaling factor is not applied to production by muons – as athe depth dependence of production by muons is much less steep than that for spallation, we deal with this by calculating P_μ at the midpoint of the sample thickness rather than applying an integrated scaling factor.

We actually do the integrations by trapezoidal integration using the function `cumtrapz`. The result of this is matching vectors of t and $N_{Xx}(t)$ for each scaling scheme. The exposure age for each scheme can then be obtained by linear interpolation of the value of t corresponding to the measured nuclide concentration.

3.1. Step size vs. accuracy.

The accuracy of trapezoidal integration for a single production pathway can be estimated as follows: For a time step with length s, extending from T to $T + s$, given a unit surface production rate, the actual nuclide concentration N developed in that time step is:

$$N_{actual} = \int_T^{T+s} e^{-\left(\lambda + \frac{\epsilon}{\Lambda}\right)t} dt \tag{9}$$

$$= \frac{-1}{\left(\lambda + \frac{\epsilon}{\Lambda}\right)} e^{-\left(\lambda + \frac{\epsilon}{\Lambda}\right)T} \left[e^{-\left(\lambda + \frac{\epsilon}{\Lambda}\right)s} - 1\right] \tag{10}$$

(11)

whereas the nuclide concentration approximated from trapezoidal integration N_{trap} is:

$$N_{trap} = \frac{s}{2}\left[e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)(T+s)} + e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)T}\right] \quad (12)$$

$$= \frac{s}{2}e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)T}\left[e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)s} + 1\right] \quad (13)$$

So the ratio of the approximate to true nuclide concentration is:

$$\frac{N_{trap}}{N_{actual}} = -\frac{s\left(\lambda+\frac{\epsilon}{\Lambda}\right)}{2}\frac{\left[e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)s}+1\right]}{\left[e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)s}-1\right]} \quad (14)$$

Several things are important here. First, this does not depend on T, so it represents the accuracy of the approximation for the entire integration as well as for a single time step. Second, this is always greater than 1, that is, trapezoidal integration always overestimates the nuclide concentration – which eventually results in an underestimate of the exposure age. Although this is accounted for by the fact that reference production rates are calibrated using the same numerical method that is used for the exposure age calculations, it is still important to minimize this inaccuracy. Third, the spallogenic part of the production-depth relationship is the limiting factor in selecting the time step – production by muons can be considered just to have larger Λ, which is always more accurate for the same s.

In practice, for an erosion rate of 10 m $\cdot$ Myr^{-1}, that is, 0.001 cm $\cdot$ yr^{-1}, which would be considered large for accurate exposure dating, the 1000-yr time step used in the calculation yields accuracy at the 10^{-5} level, which is clearly adequate for the present purpose. Note that it would not be acceptable for short-half-life radionuclides (e.g., ^{14}C) or high erosion rates.

4. Check for saturation.

If the measured nuclide concentration in the sample exceeds the highest calculated value of $N_{Xx}(t)$ for a particular scaling scheme, the sample is taken to be saturated with respect to that scaling scheme. As discussed above, the choice of the maximum length of the forward calculation means that this determination is conservative, that is, samples that are in reality close to saturation may be reported as saturated. Again, if you are interested in rigorous analysis of near-saturated samples, this code is not for you.

5. Error propagation.

We approximate the uncertainty in the exposure age results by solving the simple age equation for an effective scaling factor $S_{eff,Xx}$ for each scaling scheme Xx. This is the scaling factor that, when used in the simple age equation, yields the actual exposure age inferred from the full forward integration described above. Then the effective production rate $P_{eff,Xx} = P_{ref,Xx} \cdot S_{eff,Xx} \cdot S_{thick} \cdot S_G$. This then allows linear propagation of errors through the simple age equation to arrive at an estimate of the age uncertainty, as follows.

The internal uncertainty in the exposure age $\sigma_{int}t_{Xx}$ for scaling scheme Xx is:

$$(\sigma_{int}t_{Xx})^2 = \left(\frac{\partial t_{Xx}}{\partial N}\right)^2 \sigma N^2 \quad (15)$$

where σN is the standard error in the measured nuclide concentration and:

$$\frac{\partial t_{Xx}}{\partial N} = \left[P_{eff,Xx} - N\left(\lambda + \frac{\rho\epsilon}{\Lambda_{sp}}\right)\right]^{-1} \quad (16)$$

The external uncertainty in the exposure age $\sigma_{ext} t_{Xx}$ for scaling scheme Xx is:

$$(\sigma_{ext} t_{Xx})^2 = \left(\frac{\partial t_{Xx}}{\partial N}\right)^2 \sigma N^2 + \left(\frac{\partial t_{Xx}}{\partial P_{eff,Xx}}\right)^2 \sigma P_{eff,Xx}^2 \tag{17}$$

where

$$\frac{\partial t_{Xx}}{\partial P_{eff,Xx}} = -N \left[P_{eff,Xx}^2 - N P_{eff,Xx} \left(\lambda + \frac{\rho \epsilon}{\Lambda_{sp}}\right)\right]^{-1} \tag{18}$$

$$\sigma P_{eff,Xx} = \sigma P_{ref,Xx} * S_{thick} * S_G * S_{eff,Xx} \tag{19}$$

and $\sigma P_{ref,Xx}$ is the standard error in the reference production rate for scaling scheme Xx.

1.4 get_al_be_erosion.m

```
results = get_al_be_erosion(sample,consts,nuclide)
```

This is the main control function that carries out the erosion rate calculation. `al_be_erosion_one` and `al_be_erosion_many` call it.

The argument `sample` is a structure containing sample information. The fields are as follows:

sample.sample_name	Sample name	string
sample.lat	Latitude	double
sample.long	Longitude	double
sample.elv	elevation in meters	double
sample.pressure	pressure in hPa (optional if sample.elv is set)	double
sample.aa	Flag that indicates how to interpret the elevation value.	string
sample.thick	Sample thickness in cm	double
sample.rho	Sample density, $g \cdot cm^{-3}$	double
sample.othercorr	Shielding correction.	double
sample.N10	^{10}Be concentration, atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$	double
sample.delN10	standard error of ^{10}Be concentration	double
sample.N26	^{26}Al concentration, atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$	double
sample.delN26	standard error of ^{26}Al concentration	double

The argument `consts` is a structure containing the constants. It is typically the structure created by `make_al_be_consts_v12.m`, although only a subset of the fields in that structure are actually used by this function.

The argument `nuclide` tells the function which nuclide is being used; allowed values are 10 or 26. This is a numerical value, not a string.

This function returns a structure called `results` that contains the following fields:

Non-scaling-scheme-specific information:

results.flags	Non-fatal error messages, mostly to do with nuclide concentrations above saturation	string
results.main_version	Version number for this function	string
results.obj_version	Version of objective function called internally	string
results.mu_version	Version of P_mu_total called internally	string
results.Pmu0	Surface production rate due to muons (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1})	double

Scaling-scheme-specific information:

results.P_St	Surface production rate due to spallation (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}) following non-time-dependent Lal (1991) - Stone(2000) scaling scheme	double
results.Egcm2yr	1x5 vector containing erosion rates (g $\cdot$ cm$^2\cdot$ yr^{-1}) according to the St, De, Du, Li, and Lm scaling schemes respectively	1 x 5 double
results.EmMyr	1x5 vector containing erosion rates (m $\cdot$ Myr^{-1}) according to the St, De, Du, Li, and Lm scaling schemes respectively	1 x 5 double
results.delE_int	1x5 vector containing internal uncertainties (m $\cdot$ Myr^{-1}) according to the 5 scaling schemes. Note that these should all be essentially equal.	1 x 5 double
results.delE_ext	1x5 vector containing external uncertainties (m $\cdot$ Myr^{-1}) according to the 5 scaling schemes.	1 x 5 double

Diagnostic information:

results.fzero_status	1x5 vector containing output flags returned by rootfinding function `fzero` at solving for the erosion rates from the 5 scaling schemes.	1 x 5 double
results.fval	1x5 vector containing objective function values (atoms $\cdot$ g^{-1}) at the erosion rates from the 5 scaling schemes.	1 x 5 double
results.time_fzero	1x5 vector containing the time (s) required for each solution.	1 x 5 double
results.time_mu_precalc	Time (s) required to calculate $P_\mu(z)$.	double

1. Basic idea.

This function solves the equation:

$$\int_0^\infty \left[P_{sp,0}(t)\exp\left(-\frac{\epsilon t}{\Lambda_{sp}}\right) + P_\mu(\epsilon t)\right]\exp(-\lambda t)dt - N_m = 0 \qquad (20)$$

for the erosion rate ϵ (here in g $\cdot$ cm$^{-2}\cdot$ yr^{-1}), where Λ_{sp} is the effective attenuation length for spallogenic production (g $\cdot$ cm^{-2}), λ is the decay constant for the nuclide in question (yr^{-1}), N_m is the measured nuclide concentration (atoms $\cdot$ g^{-1}), $P_{sp,0}(t)$ is the thickness-averaged surface nuclide production rate due to spallation as a function of time (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}), and $P_\mu(z)$ is the thickness-averaged production rate due to muons (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}) as a function of the depth z (g $\cdot$ cm^{-2}) . As this cannot be solved analytically, we use the MATLAB rootfinding algorithm `fzero` to find the zero of an objective function, `ET_objective.m`, that computes the left-hand side of this equation, that is, the predicted nuclide concentration for a particular erosion rate less the measured nuclide concentration. The actual integration is described in the documentation for `ET_objective`.

2. Initial guess.

The objective function involves several numerical integrations, so finding its zero can be slow. The easiest way to speed it up is to provide it with an initial guess for the solution that is close to the actual solution. We do this by first estimating the erosion rate using the simple equation of Lal (1991), which disregards production by muons:

$$N = \frac{P}{\lambda + \frac{\epsilon}{\Lambda_{sp}}} \tag{21}$$

Here we calculate P using the non-time-dependent St scaling scheme.

This can be solved directly for ϵ to obtain the initial guess.

3. Calculate the production rates; pass to objective function; solve.

We then assemble the functions $P_{sp,0}(t)$ (for each scaling scheme) and $P_{\mu}(z)$ and pass them to the objective function, then find its zero to obtain the corresponding erosion rate for each scaling scheme.

The actual assembly of $P_{sp,0}(t)$ for the four time-dependent scaling schemes is the same as is used in `get_al_be_age.m`, and is described above in section 1.3.

$P_{\mu}(z)$ is generated on a 100-point log-spaced vector from 0 to 2×10^5 g $\cdot$ cm^{-2} using the function `P_mu_total.m`. The spacing of this vector affects the accuracy of the solution; this is discussed in more detail in the documentation for `ET_objective.m`.

4. Error propagation.

The fact that the full erosion rate equation cannot be solved analytically also makes error propagation difficult. We propagate errors linearly via the standard error propagation formula, but we cannot compute the derivatives of the derived erosion rate with respect to the uncertain input parameters, that are required in the formula, analytically, so two additional solutions of the full equation are required to estimate the partial derivative with respect to each uncertain parameter. It is time-consuming to do this using the full forward model, so we use a simplified erosion rate - nuclide concentration relationship for the uncertainty analysis:

$$\frac{P_{eff,sp}}{\lambda_i + \frac{\epsilon}{\Lambda_{sp}}} + \frac{P_{\mu}}{\lambda_i + \frac{\epsilon}{\Lambda_{\mu,eff}}} - N_m = 0 \tag{22}$$

where λ is the decay constant for the nuclide of interest, $P_{eff,sp}$ is the effective surface production rate due to spallation, P_{μ} is the surface production rate due to muons, Λ_{sp} is the effective attenuation length for production by spallation, and $\Lambda_{\mu,eff}$ is the effective attenuation length for production by muons. Both $\Lambda_{\mu,eff}$ and $P_{eff,sp}$ depend on the erosion rate, which is why this simplified equation cannot be used to accurately calculate the erosion rate in the first place. However, as we have already calculated N_{μ}, the nuclide concentration attributable to production by muons, in the course of solving the full equation above, we can calculate the value of $\Lambda_{\mu,eff}$ for which the simplified equation would yield the true erosion rate by:

$$\Lambda_{\mu,eff} = \frac{\epsilon_{true}}{\frac{P_{\mu}}{N_{\mu}} - \lambda} \tag{23}$$

where ϵ_{true} is the 'true' erosion rate calculated by solving the full equation.

In like manner, we have already calculated N_{sp}, the nuclide concentration attributable to production by spallation, so we can also calculate the value of $P_{eff,sp}$ that would yield the correct erosion rate by:

$$P_{eff,sp} = N_{sp} \left(\lambda + \frac{\epsilon_{true}}{\Lambda_{sp}} \right) \tag{24}$$

Finally, the simplified erosion rate equation cannot be solved directly either, so it is coded as an internal function that can be used as an argument to `fzero`. We can then calculate the partial derivatives of the erosion rate with respect to the uncertain parameters $\partial\epsilon/\partial N$, $\partial\epsilon/\partial P_{ref,Xx}$, etc. numerically by perturbing the input parameters and repeatedly solving the simple erosion rate equation. We use a first-order centered difference scheme.

Finally, we use the resulting partial derivatives and input parameter uncertainties to evaluate the standard error propagation formula for internal and external uncertainties. Note that the internal uncertainty, that is, the uncertainty due to measurement error alone, is for all practical purposes the same for all the different scaling schemes. On the other hand, the external uncertainties, which reflect uncertainties in production rates as well, are different for each scaling scheme.

1.5 make_al_be_consts_v2.m

Syntax:

```
make_al_be_consts_v2
```

This function has no arguments or outputs. It creates a structure called `al_be_consts` that contains all the constants relevant to the exposure age and erosion rate calculations, and saves it as a .mat file. The fields are:

1. Non-scaling-scheme-specific parameters

al_be_consts.version	Version number for this function	string
al_be_consts.prepdate	Date and time the constants structure was created	1x6 double array
al_be_consts.l10	^{10}Be decay constant (yr^{-1})	double
al_be_consts.l26	^{26}Al decay constant (yr^{-1})	double
al_be_consts.Lsp	Effective attenuation length for production by spallation (g $\cdot$ cm^{-2})	double
al_be_consts.Natoms10	number density of O atoms in quartz (atoms $\cdot$ g^{-1})	double
al_be_consts.Natoms26	number density of Si atoms in quartz (atoms $\cdot$ g^{-1})	double
al_be_consts.k_neg10	summary yield for ^{10}Be production by negative muon capture in quartz (atoms $\cdot$ (stopped μ_-)$^{-1}$)	double
al_be_consts.delk_neg10	uncertainty in above (atoms $\cdot$ (stopped μ_-)$^{-1}$)	double
al_be_consts.k_neg26	summary yield for ^{26}Al production by negative muon capture in quartz (atoms $\cdot$ (stopped μ_-)$^{-1}$)	double
al_be_consts.delk_neg26	uncertainty in above (atoms $\cdot$ (stopped μ_-)$^{-1}$)	double
al_be_consts.sigma190_10	measured cross-section at 190 GeV for ^{10}Be production from O by fast muon reactions (cm^{-2})	double
al_be_consts.delsigma190_10	uncertainty in above (cm^{-2})	double
al_be_consts.sigma190_26	measured cross-section at 190 GeV for ^{26}Al production from Si by fast muon reactions (cm^{-2})	double
al_be_consts.delsigma190_26	uncertainty in above (cm^{-2})	double

2. Scaling-scheme-specific parameters

al_be_consts.Fsp10	fraction of reference ^{10}Be production rate attributable to spallation according to Stone (2000) (nondimensional)	double
al_be_consts.Fsp26	fraction of reference ^{26}Al production rate attributable to spallation according to Stone (2000) (nondimensional)	double
al_be_consts.P10_ref_St	reference ^{10}Be production rate due to spallation for Lal(1991)/Stone(2000) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP10_ref_St	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P26_ref_St	reference ^{26}Al production rate due to spallation for Lal(1991)/Stone(2000) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP26_ref _St	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P10_ref_De	reference ^{10}Be production rate due to spallation for Desilets et al. (2006) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP10_ref_De	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P26_ref_De	reference ^{26}Al production rate due to spallation for Desilets et al. (2006) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP26_ref_De	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P10_ref_Du	reference ^{10}Be production rate due to spallation for Dunai (2001) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP10_ref_Du	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P26_ref_Du	reference ^{26}Al production rate due to spallation for Dunai (2001) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP26_ref_Du	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P10_ref_Li	reference ^{10}Be production rate due to spallation for Lifton et al. (2005) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP10_ref_Li	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P26_ref_Li	reference ^{26}Al production rate due to spallation for Lifton et al. (2005) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP26_ref_Li	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P10_ref_Lm	reference ^{10}Be production rate due to spallation for time-dependent adaptation of the Lal(1991)/Stone(2000) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP10_ref_Lm	uncertainty in above (atoms · g^{-1}· yr^{-1})	double
al_be_consts.P26_ref_Lm	reference ^{26}Al production rate due to spallation for time-dependent adaptation of the Lal(1991)/Stone(2000) scaling scheme (atoms · g^{-1}· yr^{-1})	double
al_be_consts.delP26_ref_Lm	uncertainty in above (atoms · g^{-1}· yr^{-1})	double

3.Paleomagnetic and solar variability records

al_be_consts.M	Ratio of past to present magnetic field intensity. Values from 7500-11500 yr are taken from Yang et al.; values from 12000 to 800000 are taken from SINT800; final two values (corresponding to times of 801,000 yr and Inf) are the average of the SINT800 record. (nondimensional)	1x796 double array
al_be_consts.t_M	Time vector to match above. Times are [7500:1000:11500 12000:1000:801000 Inf] (yr)	1x796 double array
al_be_consts.TTRc	3-d data block containing global maps of cutoff rigidities derived from the Korte and Constable magnetic field model by trajectory tracing (Lifton and others, in prep). Used in the De and Li scaling schemes. Dimensions are latitude, longitude, time – see below for step size in these. (GV)	37x25x15 double array
al_be_consts.IHRc	3-d data block containing global maps of cutoff rigidities derived from the Korte and Constable magnetic field model by applying Equation 2 of Dunai (2001) to inclination and horizontal field intensity values. These data are also due to Nat Lifton. Used in the Du scaling scheme. Dimensions are latitude, longitude, time – see below for step size in these. (GV)	37x25x15 double array
al_be_consts.lat_Rc	Latitude vector for interpolating cutoff rigidity data blocks. 5°spacing. (DD)	1x37 double array
al_be_consts.lon_Rc	Longitude vector for interpolating cutoff rigidity data blocks. 15°spacing. (DD)	1x25 double array
al_be_consts.t_Rc	Time vector for interpolating cutoff rigidity data blocks. Times are [0:500:6500 6900]. (yr)	1x15 double array
al_be_consts.MM0_KCL	Ratio of past to present magnetic field intensity at past times obtained by approximating the Korte and Constable magnetic field reconstruction with a geocentric dipole. Used in the Lm scaling scheme. Times match the al_be_consts.t_Rc time vector. (nondimensional)	15x1 double array
al_be_consts.lat_pp_KCL	Latitude of north magnetic pole position at past times obtained by approximating the Korte and Constable magnetic field reconstruction with a geocentric dipole. Used in the Lm scaling scheme. Times match the al_be_consts.t_Rc time vector. (nondimensional)	15x1 double array
al_be_consts.lon_pp_KCL	Longitude of north magnetic pole position at past times obtained by approximating the Korte and Constable magnetic field reconstruction with a geocentric dipole. Used in the Lm scaling scheme. Times match the al_be_consts.t_Rc time vector. (nondimensional)	15x1 double array
al_be_consts.S	Solar variability parameter used in Li scaling scheme. Resampled from data given by Lifton et al. (2005) to [0:500:6500 6900 7500:1000:10500] to match time steps in other magnetic field records. (nondimensional)	1x19 double array
al_be_consts.SInf	Long-term average solar variability parameter for use in Li scaling scheme before 10500 yr BP. See Lifton et al. (2005) for details. (nondimensional)	double

1.6 makeCplot.m

Syntax:

```
filename = makeCplot(data10,data26,localFlag)
```

This function creates the comparative exposure age plot for the single-sample exposure age results page. It makes use of numerous system calls, filenames, directories, etc. which are specific to the architecture of our web server. Thus, it is unlikely to be useful to users as a standalone function.

1.7 makeEplot.m

Syntax:

```
filename = makeEplot(pdata,localFlag)
```

This function creates the $\left[{}^{26}\mathrm{Al}\right]^* / \left[{}^{10}\mathrm{Be}\right]^* - \left[{}^{10}\mathrm{Be}\right]^*$ plot for the exposure age and erosion rate results pages. It makes use of numerous system calls, filenames, directories, etc. which are specific to the architecture of our web server. Thus, it is unlikely to be useful to users as a standalone function.

1.8 makePofTplot.m

Syntax:

```
filename = makePofTplot(tdata,localFlag)
```

This function creates the $R_C(t)$ and $P(t)$ plots for the single-sample exposure age results page. It makes use of numerous system calls, filenames, directories, etc. which are specific to the architecture of our web server. Thus, it is unlikely to be useful to users as a standalone function.

1.9 skyline_in.m

```
retstr = skyline_in(ins)
```

The MATLAB web server calls this function when the topographic shielding data input form (skyline_input.html) is submitted. It takes as input a structure containing string variables, which is supplied by the MATLAB web server. It returns a text string consisting of an output HTML document containing the results of the erosion rate calculation. The documentation for the MATLAB web server describes this process in more detail.

The input structure `ins` contains the following fields. All are string variables. These have the same names as the data-entry fields in the HTML input form.

ins.str_strike	Strike of sampled surface (degrees)
ins.str_dip	Dip of sampled surface
ins.str_az	String of space-separated azimuths
ins.str_el	String of space-separated horizon angles

Note that whole number degrees are required for all the inputs (on the basis that measurements with a greater precision than this are highly unlikely). Decimal degrees will be rejected.

The majority of this function consists of routines that check the input data to make sure it is in the expected form and is within expected bounds, and convert string variables to numerical values. After the data checking is complete, the function passes data to the function `skyline.m`, which actually carries out the erosion rate calculation and returns the result. Finally, this function assembles the output data, generates the required plots, and inserts the output data into the output HTML template.

2 Subsidiary calculation functions

2.1 angdist.m

Syntax:

```
psi = angdist(phi1,theta1,phi2,theta2)
```

This function calculates the angular distance between two points on a sphere. Given (latitude, longitude) for two points (ϕ_1, θ_1) and (ϕ_2, θ_2), the angle ψ between them is:

$$\psi = \arccos\left(cos(\phi_1)\cos(\phi_2)\left[\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)\right] + \sin(\phi_1)\sin(\phi_2)\right) \tag{25}$$

2.2 antatm.m

Syntax:

```
pressure = antatm(elevation)
```

This function converts elevation z in meters to atmospheric pressure p in hPa using the pressure-elevation relationship:

$$p(z) = 989.1 \exp\left[\frac{-z}{7588}\right] \tag{26}$$

The pressure-elevation relationship is derived from:

Radok, U., Allison, I., and Wendler, G., 1996. Atmospheric pressure over the interior of Antarctica. *Antarctic Science* 8, pp. 209-217.

For further discussion see:

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

2.3 d2r.m

Syntax:

```
radians = d2r(degrees)
```

This function converts angular measurements in degrees to angular measurements in radians.

2.4 desilets2006sp.m

Syntax:

```
scalingfactor = desilets2006sp(h,Rc)
```

Calculates the geographic scaling factor for cosmogenic-nuclide production for particular cutoff rigidity and atmospheric pressure according to the scheme in:

Desilets D., Zreda M., Prabu T., 2006. Extended scaling factors for in situ cosmogenic nuclides: New measurements at low latitude. *Earth and Planetary Science Letters*, v. 246, pp. 265-276.

The input arguments are `h`, atmospheric pressure (hPa), and `Rc`, cutoff rigidity (GV). Accepts vector arguments.

This function:

Converts atmospheric pressure h (hPa) to atmospheric depth x (g $\cdot$ cm^{-2}) by $x = 1.0197h$.

Assigns cutoff rigidities below 2 GV a value of 2 GV.

Obtains the effective attenuation length in air $\Lambda_{eff,sp}$ via Equation (4) in the source paper:

$$\Lambda_{eff,sp} = \frac{1033 - x}{g(1033, R_C) - g(x, R_C)} \tag{27}$$

where R_C is the cutoff rigidity (GV) and the function $g(x, R_C)$ is:

$$\begin{aligned} g(x, R_C) = \quad & n\left[1 + \exp\left(-\alpha R_C^{-k}\right)\right]^{-1} x && (28) \\ + \quad & \tfrac{1}{2}\left[a_0 + a_1 R_C + a_2 R_C^2\right] x^2 && (29) \\ + \quad & \tfrac{1}{3}\left[a_3 + a_4 R_C + a_5 R_C^2\right] x^3 && (30) \\ + \quad & \tfrac{1}{5}\left[a_6 + a_7 R_C + a_8 R_C^2\right] x^4 && (31) \end{aligned}$$

given constants:

n	1.0177×10^{-2}
α	1.0207×10^{-1}
k	-3.9527×10^{-1}
a_0	8.5236×10^{-6}
a_1	-6.3670×10^{-7}
a_2	-7.0814×10^{-9}
a_3	-9.9182×10^{-9}
a_4	9.9250×10^{-10}
a_5	2.4925×10^{-11}
a_6	3.8615×10^{-12}
a_7	-4.8194×10^{-13}
a_8	-1.5371×10^{-14}

Obtains the altitude scaling factor S_{alt}:

$$S_{alt} = \exp\left(\frac{1033 - x}{\Lambda_{eff,sp}}\right) \tag{32}$$

Obtains the latitude scaling factor S_{lat} via Equation (6) in the source paper:

$$S_{lat} = 1 - \exp\left(-\alpha R_C^{-k}\right) \tag{33}$$

where R_C is the cutoff rigidity, $\alpha = 10.275$, and $k = 0.9615$.

Finally, obtains the total scaling factor $S = S_{alt} S_{lat}$.

Accepts either scalars or vectors of equal sizes for all the input arguments. Returns either a scalar or a vector of the appropriate size.

2.5 dunai2001sp.m

Syntax:

```
scalingfactor = dunai2001sp(h,Rc)
```

Calculates the geographic scaling factor for cosmogenic-nuclide production for particular cutoff rigidity and atmospheric pressure according to the scheme in:

Dunai T.J., 2001. Influence of secular variation of the geomagnetic field on production rates of in situ produced cosmogenic nuclides. *Earth and Planetary Science Letters*, v. 193, pp. 197-212.

The input arguments are `h`, atmospheric pressure (hPa) and `Rc`, cutoff rigidity (GV). Accepts vector arguments.

This function:

Converts atmospheric pressure h (hPa) to atmospheric depth between sea level and the site δz ($\mathrm{g} \cdot \mathrm{cm}^{-2}$) by:

$$\delta z = 1.0197\,(1013.25 - h) \tag{34}$$

Calculates the sea level scaling factor S_{sl} according to Equation (3) of the source paper:

$$S_{sl} = Y + \frac{A}{\left[1 + \exp\left(-\frac{R_C - X}{B}\right)\right]^C} \tag{35}$$

where R_C is the cutoff rigidity (GV) and the other constants in the equation are:

A	0.5221
B	-1.7211
C	0.3345
X	4.2822
Y	0.4952

Calculates the atmospheric attenuation length Λ_{atm} according to Equation (4) of the source paper:

$$\Lambda_{atm} = y + \frac{a}{\left[1 + \exp\left(-\frac{R_C - x}{b}\right)\right]^c} \tag{36}$$

where R_C is the cutoff rigidity (GV) and the other constants in the equation are:

a	17.183
b	2.060
c	5.9164
x	2.2964
y	130.11

Finally, calculates the scaling factor at the site S according to Equation (5) of the source paper:

$$S = S_{sl} \exp\left(\frac{\delta z}{\Lambda_{atm}}\right) \tag{37}$$

Accepts either scalars or vectors of equal sizes for all the input arguments. Returns either a scalar or a vector of the appropriate size.

2.6 ET_objective.m

```
miss = ET_objective(E,cs,target,dFlag)
```

This is the objective function used by `get_al_be_erosion` to solve for the erosion rate.

The argument `E` is the erosion rate.

The argument `target` is the measured number of atoms to which the predicted value will be compared (atoms · g^{-1}). Thus, the value of `E` that yields `miss` = 0 solves the equation.

The argument `cs` is a structure with the following fields:

cs.tv	Time vector (yr)	1xn double array
cs.P_sp_t	Surface production rate due to spallation at times corresponding to those in cs.tv (atoms · g^{-1}· yr^{-1})	1xn double array
cs.z_mu	Vector of depths. This is generally [0 logspace(0,5.3,100)] plus half the sample thickness. That is, we linearize $P(z)$ within the sample to get the thickness-averaged value of P_μ that is in `cs.P_mu_z`. (g · cm^{-2})	1xm double array
cs.P_mu_z	Production rates due to muons at depths in cs.z_mu. (atoms · g^{-1}· yr^{-1})	1xm double array
cs.l	Decay constant for nuclide of interest (yr^{-1})	double
cs.tsf	Thickness scaling factor (nondimensional)	double
cs.L	Effective attenuation length for spallogenic production (g · cm^{-2})	double

The argument `dflag` is a string variable telling the function what to return. If `dflag` = 'no,' the output is just the objective function value, that is, the number of atoms difference between the measured value (`target`) and that predicted by the supplied erosion rate. If `dflag` = 'yes,' the output is a structure containing diagnostic information, as follows:

miss.ver	Version number of this function	string
miss.Nmu	Nuclide concentration in sample attributable to production by muons at the given erosion rate (atoms · g^{-1})	double
miss.Nsp	Nuclide concentration in sample attributable to production by spallation at the given erosion rate (atoms · g^{-1})	double
miss.expected_Nsp, miss.PP, miss.A, miss.tsf	Assorted diagnostics. Some are optional – generally depending on whether time-dependent or steady production is being used.	various

This function calculates the following:

$$S_{thick} \int_0^\infty P_{sp,0}(t) e^{(-\epsilon t/\Lambda_{sp})} e^{-\lambda t} dt + \int_0^\infty P_\mu(\epsilon t + \delta z/2) e^{-\lambda t} dt - N_m \quad (38)$$

where z is depth ($\mathrm{g \cdot cm^{-2}}$), δz is the sample thickness ($\mathrm{g \cdot cm^{-2}}$), ϵ is the erosion rate (the input argument `E`, here in $\mathrm{g \cdot cm^{-2} \cdot yr^{-1}}$), N_m is the measured nuclide concentration (the input argument `target`), $P_{sp}(t)$ is the surface production rate due to spallation at time t, λ is the decay constant of the nuclide of interest, Λ_{sp} is the effective attenuation length for production by spallation, S_{thick} is the thickness scaling factor (dimensionless) and $P_{\mu}(z)$ is the thickness-averaged production rate due to muons as a function of depth.

This equation is basically the predicted nuclide concentration in the sample at the given erosion rate, less the measured nuclide concentration. Thus, the correct erosion rate is the value of `E` for which this function returns zero.

The first term of this equation, that is, the predicted nuclide concentration in the sample attributable to production by spallation, is integrated semi-numerically. We divide it into timesteps to account for surface production rate changes as a function of time, use trapezoidal integration for $P_{sp}(t)$ (that is, use the average production rate during each time step in that time step) and then integrate each time step analytically. So the nuclide concentration produced in a time step that extends from T to $T+s$ is given by:

$$N_{T,T+s} = \frac{P_{sp}(T)+P_{sp}(T+s)}{2} \int_{T}^{T+s} e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)t} dt \quad (39)$$

$$= -\frac{P_{sp}(T)+P_{sp}(T+s)}{2\left(\lambda+\frac{\epsilon}{\Lambda}\right)} \left[e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)(T+s)} - e^{-\left(\lambda+\frac{\epsilon}{\Lambda}\right)T}\right] \quad (40)$$

The final time step goes to infinity.

The second term, that is, the predicted nuclide concentration in the sample attributable to production by muons, must be calculated numerically. We do this by transforming the vector of depths at which we have calculated production due to muons to a vector of times (that is, dividing by the erosion rate), then using trapezoidal integration. The upper limit of integration t_{max} is $(2 \times 10^5)/\epsilon$. The accuracy of this integration can be evaluated by similar arguments as described in the documentation for `get_al_be_age.m` above in section 1.3. The main difference here is that the effective attenuation length for production by muons increases with depth, so the step size can increase with time while still maintaining the same accuracy. Basically, the accuracy is set by the choice of the function argument `cs.z_mu`. We choose a logarithmic spacing that yields accuracy near or better than 10^{-3} for typical erosion rates.

2.7 lifton2006sp.m

Syntax:

```
scalingfactor = lifton2006sp(h,Rc,S);
```

Calculates the geographic scaling factor for cosmogenic-nuclide production for particular cutoff rigidity and altitude according to the scheme in:

Lifton N.A., Bieber J.W., Clem J.M., Duldig M.L., Evenson P., Humble J.E., Pyle R., 2005. Addressing solar modulation and long-term uncertainties in scaling secondary cosmic rays for in situ cosmogenic nuclide applications. *Earth and Planetary Science Letters*, v. 239, pp. 140-161.

The input arguments are `h`, atmospheric pressure (hPa), `Rc`, cutoff rigidity (GV), and `S`, the solar modulation factor (nondimensional; see the source paper for details). Accepts vector arguments.

This function:

Converts atmospheric pressure h (hPa) to atmospheric depth x (g $\cdot$ cm^{-2}) by multiplying by 1.0197.

Assigns cutoff rigidities below 1.907 GV a value of 1.907.

Calculates the scaling factor I (Lifton's nomenclature) using Equation (4) in the source paper:

$$\ln(I) = c_1 \ln(XS) - S \exp\left[\frac{c_2 S}{(R_C + 5S)^{2S}}\right] + c_3 X^{c_4} + c_5\left[(R_C + 4S)\,X\right]^{c_6} + c_7\,(R_C + 4S)^{c_8} \tag{41}$$

where R_C is the cutoff rigidity, S is the solar modulation factor, and the constants c_i are:

c_1	1.8399
c_2	-1.1854×10^{2}
c_3	-4.9420×10^{-2}
c_4	8.0139×10^{-1}
c_5	1.2708×10^{-4}
c_6	9.4647×10^{-1}
c_7	-3.2208×10^{-2}
c_8	1.2688

Accepts either scalars or vectors of equal sizes for all the input arguments. Returns either a scalar or a vector of the appropriate size.

2.8 NCEPatm_2.m

Syntax:

```
pressure = NCEPatm_2(site_lat,site_lon,site_elv)
```

This function converts elevation z (m) to atmospheric pressure p (hPa) using the pressure-elevation relationship in the ICAO Standard Atmosphere:

$$p(z) = p_s exp\Big\{ -\frac{gM}{R\xi} \left[\ln T_s - \ln\left(T_s - \xi z\right)\right] \Big\} \tag{42}$$

Where M is the molar weight of air, g the acceleration due to gravity, and R the gas constant, giving $gM/R = 0.03417$ $\mathrm{K} \cdot \mathrm{m}^{-1}$. The adiabatic lapse rate ξ is taken to be 0.0065 $\mathrm{K} \cdot \mathrm{m}^{-1}$.

The sea level pressure p_s (hPa) and the sea level temperature $T_s = 288.15$ K are obtained by interpolating the sample location onto global grids of annual mean sea level pressure and annual mean 1000 mbar temperature generated by the NCAR-NCEP reanalysis:

http://www.cdc.noaa.gov/ncep_reanalysis/

This function performs reasonably well for the deep southern latitudes, but it's not recommended for this purpose. The function `antatm.m` (which is fit to actual station measurements) should do a much better job.

There is more discussion of how well this atmosphere approximation performs against actual station measurements in the main text of the paper. In particular see Figures 1 and 2.

2.9 P_mu_total.m

Syntax:

```
out = P_mu_total(z,h,consts,dflag)
```

This function calculates the nuclide production rate due to muons at a particular surface elevation and depth below the surface. The method is described in:

Heisinger, B., Lal, D., Jull, A.J.T., Kubik. P., Ivy-Ochs, S., Neumaier, S., Knie, K., Lazarev, V., and Nolte, E., 2002. Production of selected cosmogenic radionuclides by muons: 1. Fast muons. *Earth and Planetary Science Letters* 200, pp. 345-355. (henceforth, H2002a).

Heisinger, B., Lal, D., Jull, A.J.T., Kubik. P., Ivy-Ochs, S., Knie, K., and Nolte, E., 2002. Production of selected cosmogenic radionuclides by muons: 2. Capture of negative muons. *Earth and Planetary Science Letters* 200, pp. 357-369. (henceforth, H2002b).

The input argument `z` is depth below the surface in $g \cdot cm^{-2}$. This argument can be a vector.

The input argument `h` is the atmospheric pressure at the surface in hPa.

The input argument consts is a structure containing nuclide-specific constants. The fields are as follows:

consts.Natoms	number density of target atoms in quartz	atoms $\cdot$ g^{-1}
consts.k_neg	summary yield for production by negative muon capture in quartz	atoms $\cdot$ (stopped μ_-)$^{-1}$
consts.sigma190	measured cross-section at 190 GeV for production by fast muon reactions	cm^{-2}

The argument `dflag` is a string variable telling the function what to return. If `dflag` = 'no,' the output is simply a vector of production rates of the same size as the input argument `z`. If `dflag` = 'yes,' the output is a structure containing diagnostic information, as follows:

out.phi_vert_slhl	Flux of vertically traveling muons at the specified depths at sea level	muons $\cdot$ cm$^{-2}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.R_vert_slhl	Stopping rate of vertically traveling muons at the specified depths at sea level	muons $\cdot$ g$^{-1}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.phi_vert_site	Flux of vertically traveling muons at the specified depths at the site elevation	muons $\cdot$ cm$^{-2}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.R_vert_site	Stopping rate of vertically traveling muons at the specified depths at the site elevation	muons $\cdot$ g$^{-1}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.phi	Total flux of muons at the specified depths at the site elevation	muons $\cdot$ cm$^{-2}\cdot$ yr^{-1}
out.R	Total stopping rate of negative muons at the specified depths at the site elevation	negative muons $\cdot$ g$^{-1}\cdot$ yr^{-1}
out.Beta	Factor describing the energy dependence of fast muon reaction cross-sections	nondimensional
out.Ebar	Mean muon energy at the specified depths	GeV
out.P_fast	Nuclide production rate by fast muon reactions at the specified depths	atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}
out.P_neg	Nuclide production rate by negative muon capture at the specified depths	atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}
out.H	Atmospheric depth between the site elevation and sea level	g $\cdot$ cm^{-2}
out.LZ	Atmospheric attenuation lengths for vertically traveling muons stopping at the specified depths	g $\cdot$ cm^{-2}

The calculation goes as follows.

1. Calculate the flux of vertically traveling muons as a function of depth at sea level and high latitude.

This is accomplished by Equations (1) and (2) in H2002a. The flux of vertically traveling muons at a depth z at sea level and high latitude $\Phi_{v,0}(z)$is:

$$\Phi_{v,0}(z) = \frac{5.401 \times 10^7}{(z+21000)\left[(z+1000)^{1.66} + 1.567 \times 10^5\right]} e^{-5.5\times10^{-6}z} \qquad (43)$$

for depths $z < 200,000$ g $\cdot$ cm^{-2}. This is Equation (1) from H2002a, modified so that z is in g $\cdot$ cm^{-2}. For greater depths, $\Phi_v(z)$ is given by Equation (2) of H2002a, similarly modified:

$$\Phi_{v,0}(z) = 1.82 \times 10^{-6} \left[\frac{121100}{z}\right]^2 e^{-\frac{z}{121100}} + 2.84 \times 10^{-13} \qquad (44)$$

The units of $\Phi_v(z)$ are muons $\cdot$ cm$^{-2}\cdot$ s$^{-1}\cdot$ sr^{-1}.

2. Calculate the stopping rate of vertically traveling muons as a function of depth at sea level and high latitude, which is equivalent to the range spectrum of vertically traveling muons at the surface.

The stopping rate of vertically traveling muons at a depth z at sea level and high latitude $R_{v,0}(z)$ is the derivative of the flux of vertically traveling muons with respect to depth. It has units of muons $\cdot$ g$^{-1}\cdot$ s^{-1}. For depths $z < 200,000$ g $\cdot$ cm^{-2},

$$R_{v,0}(z) = \frac{d}{dz}\left(\Phi_{v,0}(z)\right) = -5.401 \times 10^{7}\left[\frac{bc\frac{da}{dz} - a\left(\frac{db}{dz}c + \frac{dc}{dz}b\right)}{b^2c^2}\right] \tag{45}$$

where

$$a = e^{-5.5\times10^{-6}z} \qquad \frac{da}{dz} = -5.5\times10^{-6}e^{-5.5\times10^{-6}z} \tag{46}$$

$$b = (z + 21000) \qquad \frac{db}{dz} = 1 \tag{47}$$

$$c = (z + 1000)^{1.66} + 1.567\times10^{5} \qquad \frac{dc}{dz} = 1.66\,(z + 1000)^{0.66} \tag{48}$$

The negative sign is added because the muon flux decreases with increasing depth, so its derivative ought properly to be negative, but we would like a positive value for the stopping rate. For greater depths,

$$R_{v,0}(z) = \frac{d}{dz}\left(\Phi_{v}(z)\right) = -1.82\times10^{-6}\left[\frac{df}{dz}g + \frac{dg}{dz}f\right] \tag{49}$$

where

$$f = \left[\frac{121100}{z}\right]^2 \qquad \frac{df}{dz} = \frac{-2\,(121100)^2}{z^3} \tag{50}$$

$$g = e^{-\frac{z}{121100}} \qquad \frac{dg}{dz} = -\frac{e^{-\frac{z}{121100}}}{121100} \tag{51}$$

The stopping rate of vertically traveling muons as a function of depth is equivalent to the range spectrum of vertically traveling muons at the surface. That is, a muon that had a range of 1000 g $\cdot$ cm^{-2} at the surface will stop at a depth of 1000 g $\cdot$ cm^{-2}.

3. Adjust the range spectrum of vertically traveling muons to a different elevation.

First, calculate the difference in atmospheric pressure between sea level and the elevation of interest. We use the standard atmosphere approximation to convert elevation to atmospheric pressure. The atmospheric pressure in hPa as a function of elevation is:

$$P_{atm}(z) = P_{atm,0}exp\left(-\frac{0.03417}{0.0065}\left[\ln 288.15 - \ln\left(288.15 - 0.0065h\right)\right]\right) \tag{52}$$

where h is the elevation in meters and $P_{atm,0} = 1013.25$ hPa is the sea level pressure. Pressure can then be converted to the quantity of interest, that is, δz, the atmospheric depth in g $\cdot$ cm^{-2} between the site of interest and sea level, by $\delta z = 1.019716\,(P_{atm,0} - P_{atm}(z))$.

If the atmospheric depth between sea level and the site of interest is δz, then the vertically traveling muon flux at the surface as a function of muon range Z at sea level $R_{v,0}(Z)$ can be scaled to the vertically traveling muon flux at the surface as a function of muon range at the site $R_v(Z)$ by:

$$R_v(Z) = R_{v,0}(Z)e^{\frac{\delta z}{\Lambda_\mu(Z)}} \quad (53)$$

where $\Lambda_\mu(Z)$ is a range-dependent, that is, energy-dependent effective atmospheric attenuation length. These attenuation lengths are measured in studies of atmospheric muon fluxes. We follow H2002b and use a relation between muon momentum P and effective attenuation length $\Lambda_{mu}(P)$ derived from:

Boezio, M, and 33 co-authors, 2000. Measurement of the flux of atmospheric muons with the CAPRICE94 apparatus. *Physical Review D*, 62, 032007.

The relation is:

$$\Lambda_{mu}(P) = 263 + 150P \quad (54)$$

In order to use this relation, we convert muon range to momentum using tabulated values in:

Groom, D.E., Mokhov, N.V., and Striganov, S.I., 2001. Muon stopping power and range tables 10 MeV - 100 TeV. *Atomic data and nuclear data tables* 78, pp. 183-356.

Having obtained Λ_{mu} for muons stopping at the depths of interest, we can then obtain the range spectrum of vertically traveling muons at the surface at the site of interest, $R_v(Z)$, which is equivalent to the muon stopping rate as a function of depth at the site of interest $R_v(z)$, by applying Equation 53.

4. Latitudinal variability in the range spectrum.

Although the muon range spectrum is expected to change with latitude as well as elevation due to geomagnetic effects, this effect is expected to be small. We follow H2002b and ignore it.

5. Calculate the flux of vertically traveling muons at the site of interest as a function of depth.

The flux of vertically traveling muons as a function of depth at the site of interest $\phi_v(z)$ is the integral of the muon stopping rate as a function of depth at the site of interest $R_v(z)$, which we have just calculated, from infinite depth to depth z. That is, the muon flux at a particular depth is composed of all the muons which stop below that depth. Thus, the flux of vertically traveling muons as a function of depth at the site of interest $\phi_v(z)$ is:

$$\phi_v(z) = \int_z^\infty R_v(x)dx \quad (55)$$

We actually do this integral numerically, although it would be possible to obtain an analytical expression. We take the limit of integration to be 2×10^5 g $\cdot$ cm^{-2}, where the muon flux is essentially negligible for our purposes.

6. Calculate the total muon flux as a function of depth at the site of interest.

Following equations (3) - (5) in H2002a, the zenith angle dependence of the muon flux is:

$$\phi(z,\theta) = \phi_v(z)cos^{n(z)}\theta \quad (56)$$

where θ is the zenith angle and n is given by Equation (4) in H2002a, modified so that z is given in g $\cdot$ cm^{-2}:

$$n(z) = 3.21 - 0.297 \ln\left(\frac{z}{100} + 42\right) + 1.21 \times 10^{-5} z \tag{57}$$

The total muon flux at a particular depth $\phi(z)$ then consists of Equation 56 integrated over the entire upper hemisphere and has units of muons $\cdot$ cm$^{-}2\cdot$ s^{-1}. This is given by:

$$\phi(z) = \frac{2\pi}{n(z+\delta z)+1}\phi_v(z) \tag{58}$$

7. Calculate the total muon stopping rate as a function of depth at the site of interest.

The total muon stopping rate as a function of depth Rz is the derivative with respect to depth of the total muon flux as a function of depth $\phi(z)$, that is:

$$\begin{aligned}
R(z) &= \frac{d}{dz}\left(\phi(z)\right) &(59)\\
&= \frac{d}{dz}\left(\frac{2\pi}{n(z+\delta z)+1}\phi_v(z)\right) &(60)\\
&= \frac{2\pi}{n(z+\delta z)+1}\frac{d}{dz}\left(\phi_v(z)\right) - \phi_v(z)\frac{d}{dz}\left(\frac{2\pi}{n(z+\delta z)+1}\right) &(61)\\
&= \frac{2\pi}{n(z+\delta z)+1}R_v(z) - \phi_v(z)\left(-2\pi\right)\left(n(z+\delta z)+1\right)^2\frac{d}{dz}\left(n(z+\delta z)+1\right) &(62)\\
&= \frac{2\pi}{n(z+\delta z)+1}R_v(z) - \phi_v(z)\left(-2\pi\right)\left(n(z+\delta z)+1\right)^2\left[\frac{-0.297\times 10^{-2}}{\frac{z+\delta z}{100}+42} + 1.21\times 10^{-5}\right] &(63)
\end{aligned}$$

which, as we have already calculated $n(z)$, $R_v(z)$, and $\phi_v(z)$, we can calculate easily. Again, a factor of -1 is added to obtain a positive number of stopped muons.

To summarize, we have now calculated the total muon flux $\phi(z)$ (muons $\cdot$ cm$^{-2}\cdot$ s $^{-1}$) and the total stopping rate of muons $R(z)$ (muons $\cdot$ g$^{-1}\cdot$ s $^{-1}$) at our site. We convert these to muons $\cdot$ cm$^{-2}\cdot$ yr $^{-1}$ and muons $\cdot$ g$^{-1}\cdot$ yr $^{-1}$, respectively, by multiplying them by 3.154×10^7 s $\cdot$ yr^{-1}.

Finally, we compute the stopping rate of negative muons $R^{-}(z) = 0.44R(z)$ (negative muons $\cdot$ g$^{-1}\cdot$ yr $^{-1}$).

8. Calculate the nuclide production rate due to negative muon capture.

Following Equation (11) in H2002b, the production rate of nuclide i (atoms $\cdot$ g $^{-1}\cdot$ yr^{-1}) from negative muon capture $P_{i,\mu-}(z)$ is:

$$P_{i,\mu-}(z) = R^{-}(z) f_{i,C} f_{i,D} f_i^* \tag{64}$$

where

$$f_{10,C} = 0.704 \qquad f_{26,C} = 0.296 \tag{65}$$
$$f_{10,D} = 0.1828 \qquad f_{26,D} = 0.6559 \tag{66}$$
$$f^*_{10} = 0.0043 \qquad f^*_{26} = 0.022 \tag{67}$$

9. Calculate the nuclide production rate due to fast muon reactions.

Following Equation (14) in H2002a, the production rate of nuclide i (atoms · g $^{-1}$· yr^{-1}) from fast muon interactions $P_{i,\mu fast}(z)$ is:

$$P_{i,\mu fast}(z) = \phi(z)\sigma_{0,i}\beta(z)\left(\bar{E}(z)\right)^{\alpha} N_{t,i} \tag{68}$$

where $\alpha = 0.75$.

$\sigma_{0,i}$ is the nominal zero-energy muon interaction cross-section for the reaction responsible for producing nuclide i. Here is has units of cm^{-2}. The muon interaction cross-section for a particular reaction is thought to depend on the muon energy as follows:

$$\sigma_i(E) = \sigma_{0,i}E^{\alpha} \tag{69}$$

$\sigma_{0,i}$ is determined from the measured cross-sections $\sigma_i(E)$ at 190 GeV energy in Table 1 of H2002a using this equation. These values for ^{10}Be and ^{26}Al are:

$$\sigma_{0,10} = \frac{0.094 \times 10^{-27}}{190^{\alpha}} \qquad \sigma_{0,26} = \frac{1.41 \times 10^{-27}}{190^{\alpha}} \tag{70}$$

$N_{t,i}$ is the number density of atoms of the target element (atoms · g^{-1}). The values for O and Si relevant to ^{10}Be and ^{26}Al production respectively are:

$$N_{t,10} = 2.006 \times 10^{22} \qquad N_{t,26} = 1.003 \times 10^{22} \tag{71}$$

where β is a function of depth and is approximated by Equation (16) of H2002a:

$$\beta(z) = 0.846 = 0.015 \ln\left(\frac{z}{100} + 1\right) + 0.003139 \left[\ln\left(\frac{z}{100}\right)\right] \tag{72}$$

and $\bar{E}(z)$ is the mean muon energy at depth z and is given by Equation (11) of H2002a:

$$\bar{E}(z) = 7.6 + 321.7 \left(1 - e^{-8.059\times10^{-6}z}\right) + 50.7(1 - e^{-5.05\times10^{-7}z}) \tag{73}$$

2.10 r2d.m

Syntax:

```
degrees = r2d(radians)
```

This function converts angular measurements in radians to angular measurements in degrees.

2.11 skyline.m

Syntax:

```
[out,data,ver] = skyline(az,el,strike,dip)
```

This function calculates the topographic shielding correction to the cosmic-ray flux at a site whose horizon is obstructed either by a dipping surface or by the surrounding topography.

The input arguments `strike` and `dip` are the strike and dip of the sampled surface in degrees.

The input arguments `az` and `el` are vectors containing the azimuths (degrees; $0 <$ az < 360; north = 0) and horizon angles (degrees; $0 <$ el < 360; zenith = 90) of points on the horizon.

`out` is the shielding correction for the sample (nondimensional; the ratio of the nuclide production rate at the shielded site to the nuclide production rate at an unshielded site at the same location).

`data` is a vector containing the interpolated horizon angle visible to the sample, in degrees, for 1-degree increments of azimuth. It can be used to plot the horizon, e.g., `plot(0:360,data)`

`ver` is a string variable containing the version number of the function.

The calculation goes as folllows:

First, we divide the range of azimuths $0 < \phi < 2\pi$ into 1-degree increments.

Second, we calculate the horizon angle θ of the dipping surface as a function of azimuth:

$$\theta = \arctan\left[\tan\theta_d \cos\left(\phi - \phi_s\right)\right] \tag{74}$$

where θ_d is the dip and ϕ_d is the strike of the surface.

Third, we interpolate the supplied horizon points to give the horizon angle of the topography as a function of azimuth.

Fourth, at each increment of azimuth, we take the higher of the horizon angle due to the dipping surface or the horizon angle due to the surrounding topography to be the horizon angle visible to the sample. This results in a vector $\theta(\phi)$ which defines the visible horizon angle θ at each increment of azimuth ϕ.

Finally, the fraction of the cosmic-ray flux $f(\phi)$ that lies below the horizon angle in the increment of azimuth at ϕ is:

$$f(\phi) = \frac{\delta\phi}{2\pi}\left(\sin\theta\right)^{3.3} \tag{75}$$

where $\delta\phi$ is the width of the azimuth increment in radians ($\pi/180$). The total fraction of the cosmic-ray flux that lies below the horizon f is the sum of the values of $f(\phi)$ for each increment of ϕ. The topographic shielding correction is then $(1 - f)$.

2.12 stdatm.m

Syntax:

```
pressure = stdatm(elevation)
```

This function converts elevation $z$in meters to atmospheric pressure p in hPa using the pressure-elevation relationship in the ICAO Standard Atmosphere:

$$p(z) = p_s exp\Big\{ - \frac{gM}{R\xi} \left[\ln T_s - \ln \left(T_s - \xi z \right) \right] \Big\} \tag{76}$$

where sea level pressure $P_s = 1013.25$ hPa; sea level temperature $T_s = 288.15$ K; and the adiabatic lapse rate $\xi = 0.0065$ K $\cdot$ m^{-1}. M is the molar weight of air, g the acceleration due to gravity, and R the gas constant, giving $gM/R = 0.03417$ K $\cdot$ m^{-1}.

For further discussion see:

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

2.13 stone2000.m

Syntax:

```
scalingfactor = stone2000(latitude, pressure, fsp)
```

Calculates the geographic scaling factor for cosmogenic-nuclide production with varying latitude and altitude according to the scheme in:

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

The input arguments are `latitude`, the site latitude (decimal degrees), `pressure`, the atmospheric pressure at the site (hPa), and `fsp` the fraction of production due to neutron spallation (nondimensional). Accepts vector arguments. The argument `fsp`, if omitted, defaults to 0.978, which is the correct value for ^{10}BeṪhe corresponding value for ^{26}Al is 0.974. Note that using `fsp` = 0.844 (^{10}Be) and `fsp` = 0.844 (^{26}Al) will closely reproduce the scaling factors given in Lal (1991) as long as the standard atmosphere is used. Note also that this function can be made to yield the scaling factor for spallation only by specifying `fsp` = 1 and the scaling factor for production by muons only by specifying `fsp` = 0.

The geographic production rate scaling factor S_{geo} is:

$$S_{geo}(t) = f_{sp}S_{sp} + (1 - f_{sp})S_{\mu} \tag{77}$$

where f_{sp} is the fraction of production due to spallation (input argument `fsp`), S_{sp} is the scaling factor for production by neutron spallation:

$$S_{sp}(p) = a + b\exp\frac{-p}{150} + cp + dp^2 + ep^3 \tag{78}$$

and S_{μ} is the scaling factor for production by muons:

$$S_{i,\mu}(p) = M_{sl}\exp\left(\frac{1013.25 - p}{242}\right) \tag{79}$$

Where the constants $a...e$ and M_{sl} depend on the latitude and are defined for certain index latitudes:

Latitude	a	b	c	d	e	M_{sl}
0°	31.8518	250.3193	-0.083393	7.4260e-5	-2.2397e-8	0.587
10°	34.3699	258.4759	-0.089807	7.9457e-5	-2.3697e-8	0.600
20°	40.3153	308.9894	-0.106248	9.4508e-5	-2.8234e-8	0.678
30°	42.0983	512.6857	-0.120551	1.1752e-5	-3.8809e-8	0.833
40°	56.7733	649.1343	-0.160859	1.5463e-5	-5.0330e-8	0.933
50°	69.0720	832.4566	-0.199252	1.9391 e-5	-6.3653e-8	1.000
$>$ 60°	71.8733	863.1927	-0.207069	2.0127e-5	-6.6043e-8	1.000

This table duplicates Table 1 in Stone (2000).

This function actually calculates S_{sp} and S_{mu} for index latitudes that bound the actual latitude of the site, and then determines the value at the site by linear interpolation. The exact method of interpolation used is relatively unimportant, but we have chosen linear interpolation to avoid the problem of under- and over-shooting between index latitudes due to the polynomial form of the Lal equations. Basically, linear interpolation guarantees that the scaling factor at the site is not outside the range of the scaling factors at the bounding index latitudes.

2.14 stone2000Rcsp.m

Syntax:

```
scalingfactor = stone2000Rcsp(h,Rc)
```

Calculates the geographic scaling factor for cosmogenic-nuclide production with varying cutoff rigidity and atmospheric pressure. Uses an adaptation of the scheme in Stone (2000) and Lal (1991) that allows for higher values of cutoff rigidity than were permitted by the original scheme.

Relevant papers are:

Lal D., 1991. Cosmic ray labeling of erosion surfaces: *in situ* nuclide production rates and erosion models. *Earth and Planetary Science Letters*, v. 104, pp. 424-439.

Nishiizumi K., Winterer E.L., Kohl C.P., Klein J., Middleton R., Lal D., Arnold J.R., 1989. Cosmic ray production rates of ^{10}Be and ^{26}Al from glacially polished rocks. *Journal of Geophysical Research* 94. pp. 17,907-17.915.

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

The input arguments are `h`, atmospheric pressure (hPa) and `Rc`, cutoff rigidity (GV). Accepts either scalars or vectors of equal sizes for all the input arguments. Returns either a scalar or a vector of the appropriate size.

The function:

First, uses Equation (2) of Stone (2000) to calculate the scaling factor for spallation $S(R_C, h)$ for the site atmospheric pressure, at index values of cutoff rigidity R_C corresponding to the index latitudes used in the source paper. This equation is:

$$S(R_C, h) = a + b \exp \frac{-h}{150} + ch + dh^2 + eh^3 \tag{80}$$

Where the constants $a...e$ are defined for certain index cutoff rigidities:

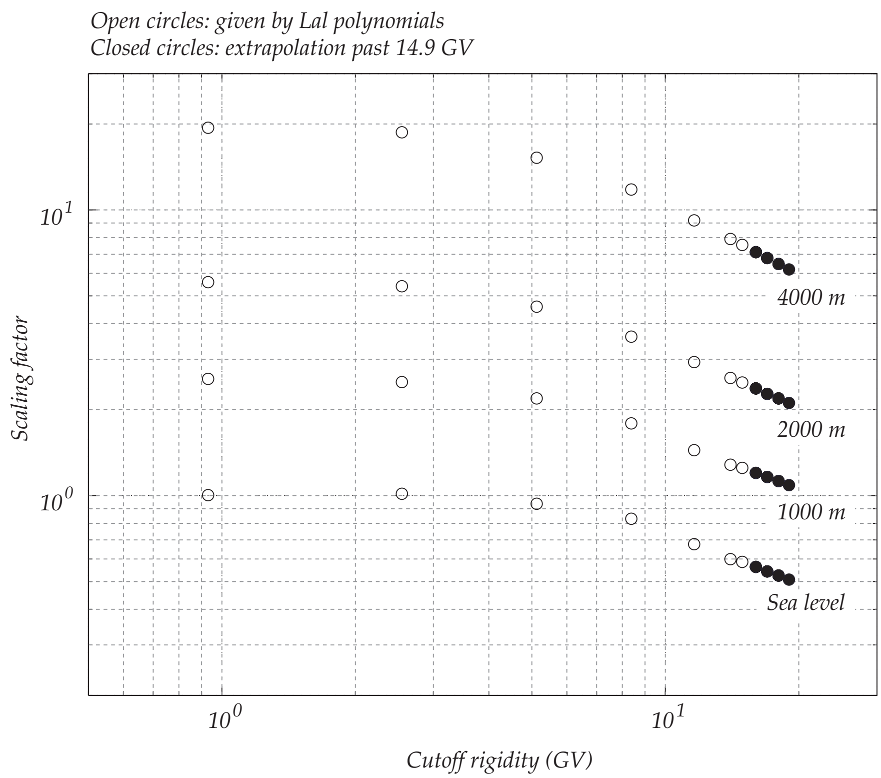
R_C	a	b	c	d	e
14.9	31.8518	250.3193	-0.083393	7.4260e-5	-2.2397e-8
14.02	34.3699	258.4759	-0.089807	7.9457e-5	-2.3697e-8
11.62	40.3153	308.9894	-0.106248	9.4508e-5	-2.8234e-8
8.38	42.0983	512.6857	-0.120551	1.1752e-5	-3.8809e-8
5.13	56.7733	649.1343	-0.160859	1.5463e-5	-5.0330e-8
2.54	69.0720	832.4566	-0.199252	1.9391 e-5	-6.3653e-8
$<$ 0.93	71.8733	863.1927	-0.207069	2.0127e-5	-6.6043e-8

The index values of cutoff rigidity are simply the index latitudes given in the source paper, latitude θ having been transformed to cutoff rigidity R_C by:

$$R_C = 14.9 \cos^4(\theta) \tag{81}$$

This is essentially the same as the method of correcting for paleomagnetic variation given in Nishiizumi et al. (1989), which is based on Equation 81.

Second, obtains the scaling factor at the particular value of R_C at the site from the scaling factor - cutoff rigidity pairs at the index rigidities by linear interpolation. For cutoff rigidities greater than 14.9 GV, this requires extending the S- R_C relationship to higher R_C. We do this by regressing the the highest three values of $\log R_C$ against the corresponding values of $\log S$ for the site atmospheric pressure, then using the resulting line in log-log space to extrapolate the scaling factor-rigidity relationship past 14.9 GV. Here is an example of the results of this procedure at various altitudes:



Basically we are assuming that the scaling factor decreases log-linearly with the cutoff rigidity at rigidities greater than 10 GV. This is a common first-order approximation in cosmic-ray physics texts – see, for example, Sandstrom. The important thing about this assumption is that in practice it is very rarely used – it is only necessary for low latitudes for several thousand years in the late Holocene – so whether it is or is not exact has minimal importance to the eventual result of nearly any exposure-age calculation.

We use log-log linear regression on the highest three R_C values, instead of a direct extension of the highest two values, to smooth irregularities that result from the polynomial form of the Lal scaling factor curves.

2.15 thickness.m

Syntax:

```
Sthick = thickness(zmax, Lambdasp, rho)
```

Calculates the thickness correction factor for nuclide production by spallation. The arguments are `zmax`, the sample thickness in cm; `Lambdasp`, the effective attenuation length for production by neutron spallation in atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}; and `rho`, the sample density in g $\cdot$ cm^{-3}. Accepts vector inputs.

Calculating the thickness correction factor S_{thick} begins by assuming that the production rate decreases with depth according to a single exponential function:

$$P(z) = P(0)\exp(\frac{\rho z}{\Lambda_{sp}}) \tag{82}$$

where $P(0)$ is the surface production rate and z is depth below the surface (cm). The thickness scaling factor S_{thick} is then:

$$S_{thick} = \frac{1}{z_{max}}\int_0^{z_{max}} P(z)dz = \frac{\Lambda_{sp}}{\rho z_{max}}\left[1 - \exp(\frac{-\rho z_{max}}{\Lambda_{sp}})\right] \tag{83}$$

where Λ_{sp} is the effective attenuation length for production by spallation (input argument `Lambdasp`) and ρ is the sample density (input argument `rho`).

3 Plotting functions

3.1 ellipse.m

Syntax:

```
[x,y] = ellipse(N10,delN10,N26,delN26,0,plotString)
[h1] = ellipse(N10,delN10,N26,delN26,1,plotString)
[h1,h2] = ellipse(N10,delN10,N26,delN26,2,plotString)
```

Plots an uncertainty ellipse on the [^{10}Be]* – [^{26}Al]* / [^{10}Be]* diagram (the 'Lal-Klein-Nishiizumi Al-Be diagram').

The arguments `N10`, `delN10`, `N26`, and `delN26` are the ^{10}Be and ^{26}Al concentrations and their uncertainties (atoms · g^{-1}).

The third argument can be 0, in which case the function returns the x and y coordinates of the $1-\sigma$ uncertainty ellipse; 1, in which case the function plots the 1σ uncertainty ellipse and returns its handle; or 2, in which case the function plots 1σ and 2σ uncertainty ellipses and returns their handles. The default is 1.

The argument `plotString` is an optional string telling the function what line type and color to use to draw the ellipse. See the documentation for the MATLAB function `plot` for more information.

What this function actually does is calculate the joint probability on a mesh in (x, y):

$$p(x,y) = x \exp\left(-0.5\left[\frac{xy - N_{26}}{\sigma N_{26}}\right]^2 + \left[\frac{x - N_{10}}{\sigma N_{10}}\right]^2\right) \tag{84}$$

It then creates a normalized cumulative distribution for p to determine the values of p that account for 68% and 95% of the total probability, and draws contours of the 2-d PDF at those values.