CRONUS-Earth ^{26}Al-^{10}Be exposure age calculator MATLAB function reference

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Appendix I of Balco, G., and Stone, J., A simple,
internally consistent, and easily accessible means of calculating surface
exposure ages or erosion rates from ^{10}Be and ^{26}Al measurements.

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Syntax:

```
retstr = al_be_age_one(ins)
```

The MATLAB web server calls this function when the exposure age data input form is submitted. It takes as input a structure containing string variables, which is supplied by the MATLAB web server. It returns a text string consisting of an output HTML document containing the results of the exposure age calculation. The documentation for the MATLAB web server describes this process in more detail.

The input structure `ins` contains the following fields. These have the same names as the data-entry fields in the HTML input form.

ins.sample_name	Sample name
ins.str_lat	Latitude
ins.str_long	Longitude
ins.str_alt	Either elevation in meters or pressure in hPa
ins.aa	Flag that indicates how to interpret the str_alt variable. Has three possible values: 'std' if the elevation value is in meters and the standard atmosphere approximation is to be used; 'ant' if the elevation value is in meters and the Antarctic atmosphere approximation is to be used; and 'pre' if the elevation value is in hPa.
ins.str_thick	Sample thickness in cm
ins.str_rho	Sample density, $g \cdot cm^{-3}$
ins.str_othercorr	Shielding correction. The shielding correction can be calculated using `skyline.m`
ins.str_E	Erosion rate, $cm \cdot yr^{-1}$
ins.str_N10	^{10}Be concentration, atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$
ins.str_delN10	standard error of ^{10}Be concentration
ins.str_N26	^{26}Al concentration, atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$
ins.str_delN26	standard error of ^{26}Al concentration

The majority of this function consists of routines that check the input data to make sure it is in the expected form and is within expected bounds, and convert string variables to numerical values. After the data checking is complete, the function then assembles the data sets needed for the ^{26}Al and ^{10}Be exposure age calculations, loads the data file containing values for physical constants, and passes data to the function `get_al_be_age.m`, which actually carries out the exposure age calculation and returns the result. Finally, this function assembles the output data, generates the required plots, and inserts the output data into the output HTML template.

The only actual calculation that takes place inside this function is the calculation of the uncertainty in the ^{26}Al ^{10}Be ratio. Denote the ^{26}Al/^{10}Be ratio by $R_{26/10}$ and its 1σ uncertainty by $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$(\sigma R_{26/10})^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2 \tag{1}$$

where N_i is the concentration of nuclide i and σN_i is its 1σ analytical uncertainty.

1.2 al_be_age_many.m

Syntax:

```
retstr = al_be_age_many(ins)
```

This function is essentially the same as `al_be_age_one.m`, with the exception that the input structure has only one field, the text string which the user pasted into the text input block in the multiple-sample input form. Thus, the input data checking and parsing routines are slightly different.

The formatting rules for entering a text block containing data for multiple samples are as follows:

1. Enter plain ASCII text only.
2. Each sample should occupy its own line.
3. Each line should have thirteen elements, as described below.
4. Elements should be separated from each other by white space (spaces or tabs).
5. Something other than white space must be entered for each element. For example, if you have no ^{26}Al measurements for a sample, you must enter '0' in the ^{26}Al concentration and ^{26}Al uncertainty positions.

In most cases, pasting directly from an Excel spreadsheet should satisfy the rules. An example of an acceptable input text block appears below.

The thirteen elements are as follows. These are the same as the input parameters on the single-sample form.

1. Sample name. Any text string not exceeding 24 characters. Sample names may not contain white space or any characters that could be interpreted as delimiters or escape characters, e.g., slashes of both directions, commas, quotes, colons, etc. Stick to letters, numbers, and dashes.
2. Latitude. Decimal degrees.
3. Longitude. Decimal degrees.
4. Elevation/pressure. Meters or hPa, respectively, depending on selection below.
5. Elevation/pressure flag. Specifies how to treat the elevation/pressure value. This is a three-letter text string. If you have supplied elevations in meters and the standard atmosphere is applicable at your site (locations outside Antarctica), enter 'std' here. If you have supplied elevations in meters and your site is in Antarctica, enter 'ant' here. If you have entered pressure in hPa, enter 'pre' here. Any text other than these three options will be rejected.
6. Sample thickness. Centimeters.
7. Sample density. $g \cdot cm^{-3}$.
8. Shielding correction. Samples with no topographic shielding, enter 1. For shielded sites, enter a number between 0 and 1. The shielding correction can be calculated using `skyline.m`.
9. Erosion rate inferred from independent evidence. $cm \cdot yr^{-1}$.
10. ^{10}Be concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.

11. Uncertainty in ^{10}Be concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.
12. ^{26}Al concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.
13. Uncertainty in ^{26}Al concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.

Here is an example of an acceptable input text block:

```
PH-1 41.3567 -70.7348 91 std 4.5 2.7 1 8e-5 123500 3700 712400 31200
01-MBL-059-BBD -77.073 -145.686 712 ant 4.75 2.65 0.997 0 0 0 1.9e6 4.9e4
NH-1 57.968 -6.812 790 std 3 2.65 1 0 943000 28000 0 0
```

As with `al_be_age_one.m`, the only calculation that takes place inside this function is the calculation of the uncertainty in the ^{26}Al ^{10}Be ratio. Denote the ^{26}Al/^{10}Be ratio by $R_{26/10}$ and its 1σ uncertainty by $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$\left(\sigma R_{26/10}\right)^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2 \tag{2}$$

where N_i is the concentration of nuclide i and σN_i is its 1σ analytical uncertainty.

1.3 al_be_erosion_one.m

Syntax:

```
retstr = al_be_erosion_one(ins)
```

The MATLAB web server calls this function when the erosion rate data input form is submitted. It takes as input a structure containing string variables, which is supplied by the MATLAB web server. It returns a text string consisting of an output HTML document containing the results of the erosion rate calculation. The documentation for the MATLAB web server describes this process in more detail.

The input structure `ins` contains the following fields. These have the same names as the data-entry fields in the HTML input form.

ins.sample_name	Sample name
ins.str_lat	Latitude
ins.str_long	Longitude
ins.str_alt	Either elevation in meters or pressure in hPa
ins.aa	Flag that indicates how to interpret the str_alt variable. Has three possible values: 'std' if the elevation value is in meters and the standard atmosphere approximation is to be used; 'ant' if the elevation value is in meters and the Antarctic atmosphere approximation is to be used; and 'pre' if the elevation value is in hPa.
ins.str_thick	Sample thickness in cm
ins.str_rho	Sample density, $g \cdot cm^{-3}$
ins.str_othercorr	Shielding correction. The shielding correction can be calculated using `skyline.m`
ins.str_N10	^{10}Be concentration, atoms $\cdot\, g^{-1} \cdot\, yr^{-1}$
ins.str_delN10	standard error of ^{10}Be concentration
ins.str_N26	^{26}Al concentration, atoms $\cdot\, g^{-1} \cdot\, yr^{-1}$
ins.str_delN26	standard error of ^{26}Al concentration

The majority of this function consists of routines that check the input data to make sure it is in the expected form and is within expected bounds, and convert string variables to numerical values. After the data checking is complete, the function then assembles the data sets needed for the ^{26}Al and ^{10}Be exposure age calculations, loads the data file containing values for physical constants, and passes data to the function `get_al_be_erosion.m`, which actually carries out the erosion rate calculation and returns the result. Finally, this function assembles the output data, generates the required plots, and inserts the output data into the output HTML template.

The only actual calculation that takes place inside this function is the calculation of the uncertainty in the ^{26}Al ^{10}Be ratio. Denote the ^{26}Al/^{10}Be ratio by $R_{26/10}$ and its 1σ uncertainty by $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$(\sigma R_{26/10})^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2 \tag{3}$$

where N_i is the concentration of nuclide i and σN_i is its 1σ analytical uncertainty.

1.4 al_be_erosion_many.m

Syntax:

```
retstr = al_be_erosion_many(ins)
```

This function is essentially the same as `al_be_erosion_one.m`, with the exception that the input structure has only one field, the text string which the user pasted into the text input block in the multiple-sample input form. Thus, the input data checking and parsing routines are slightly different.

The formatting rules for entering a text block containing data for multiple samples are as follows:

1. Enter plain ASCII text only.
2. Each sample should occupy its own line.
3. Each line should have twelve elements, as described below.
4. Elements should be separated from each other by white space (spaces or tabs).
5. Something other than white space must be entered for each element. For example, if you have no ^{26}Al measurements for a sample, you must enter '0' in the ^{26}Al concentration and ^{26}Al uncertainty positions.

In most cases, pasting directly from an Excel spreadsheet should satisfy the rules. An example of an acceptable input text block appears below.

The thirteen elements are as follows. These are the same as the input parameters on the single-sample form.

1. Sample name. Any text string not exceeding 24 characters. Sample names may not contain white space or any characters that could be interpreted as delimiters or escape characters, e.g., slashes of both directions, commas, quotes, colons, etc. Stick to letters, numbers, and dashes.
2. Latitude. Decimal degrees.
3. Longitude. Decimal degrees.
4. Elevation/pressure. Meters or hPa, respectively, depending on selection below.
5. Elevation/pressure flag. Specifies how to treat the elevation/pressure value. This is a three-letter text string. If you have supplied elevations in meters and the standard atmosphere is applicable at your site (locations outside Antarctica), enter 'std' here. If you have supplied elevations in meters and your site is in Antarctica, enter 'ant' here. If you have entered pressure in hPa, enter 'pre' here. Any text other than these three options will be rejected.
6. Sample thickness. Centimeters.
7. Sample density. $g \cdot cm^{-3}$.
8. Shielding correction. Samples with no topographic shielding, enter 1. For shielded sites, enter a number between 0 and 1. The shielding correction can be calculated using `skyline.m`.
9. ^{10}Be concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.
10. Uncertainty in ^{10}Be concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.

11. ^{26}Al concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.
12. Uncertainty in ^{26}Al concentration. Atoms $\cdot$ g^{-1}. Standard or scientific notation.

Here is an example of an acceptable input text block:

```
FV-TOP-1 38.6139 -109.1878 2527 std 2 2.5 1 4.59e5 1.3e4 2.69e6 8.80e4
FV-TOP-2 38.6136 -109.1955 2580 std 2 2.5 1 202000 8000 1100000 49000
FV-TOP-3 38.6204 -109.2062 2592 std 2 2.5 1 1.02e6 2.60e4 6.05e6 1.31e5
04-AV-PIT9-NEW -77.8282 160.9762 1300 ant 2 1.9 0.973 6710000 116000 0 0
```

As with `al_be_erosion_one.m`, the only calculation that takes place inside this function is the calculation of the uncertainty in the ^{26}Al ^{10}Be ratio. Denote the ^{26}Al/^{10}Be ratio by $R_{26/10}$ and its 1σ uncertainty by $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$\left(\sigma R_{26/10}\right)^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2 \tag{4}$$

where N_i is the concentration of nuclide i and σN_i is its 1σ analytical uncertainty.

1.5 get_al_be_age.m

```
results = get_al_be_age(sample,consts,nuclide)
```

This is the main control function that carries out the exposure age calculation. `al_be_age_one` and `al_be_age_many` call it.

The argument `sample` is a structure containing sample information. The fields are as follows:

sample.sample_name	Sample name	string
sample.lat	Latitude	double
sample.long	Longitude	double
sample.elv	elevation in meters	double
sample.pressure	pressure in hPa (optional if sample.elv is set)	double
sample.aa	Flag that indicates how to interpret the elevation value.	string
sample.thick	Sample thickness in cm	double
sample.rho	Sample density, $g \cdot cm^{-3}$	double
sample.othercorr	Shielding correction.	double
sample.E	Erosion rate, $cm \cdot yr^{-1}$	double
sample.N10	^{10}Be concentration, atoms $\cdot\ g^{-1} \cdot yr^{-1}$	double
sample.delN10	standard error of ^{10}Be concentration	double
sample.N26	^{26}Al concentration, atoms $\cdot\ g^{-1} \cdot yr^{-1}$	double
sample.delN26	standard error of ^{26}Al concentration	double

The argument `consts` is a structure containing the constants. It is typically the structure created by `make_al_be_consts_v12.m`, although only a subset of the fields in that structure are actually used by this function.

The argument `nuclide` tells the function which nuclide is being used; allowed values are 10 or 26. This is a numerical value, not a string.

This function returns a structure called `results` that contains the following fields:

results.main_version	Version number for this function	string
results.simplet	Exposure age (yr)	double
results.delt_int	Internal uncertainty (yr)	double
results.delt_ext	External uncertainty (yr)	double
results.thick_sf	Thickness correction	double
results.simple_sf	Geographic scaling factor	double
results.Psp	Thickness-integrated surface production rate due to spallation	double
results.Pmu	Thickness-integrated surface production rate due to muons	double

The exposure age calculation goes as follows:

Calculate the thickness scaling factor S_{thick} by calling the function `thickness.m`.

If `sample.pressure` is not set, calculate it by calling either `stdatm.m` or `antatm.m`.

Calculate the geographic scaling factor $S_{i,geo}$ for nuclide i by calling the function `stone2000.m`.

The production rate of nuclide i in the sample P_i (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}) is :

$$P_i = P_{i,ref} * S_{thick} * S_T * S_{i,geo} \tag{5}$$

where $P_{i,ref}$ is the reference production rate for nuclide i and S_T is the topographic shielding correction.

The exposure age t_i for nuclide i is then:

$$t_i = \frac{1}{\lambda_i + \frac{\rho\epsilon}{\Lambda_{sp}}} \ln\left[1 - \frac{N_i}{P_i}\left(\lambda_i + \frac{\rho\epsilon}{\Lambda_{sp}}\right)\right] \tag{6}$$

where N_i is the measured concentration of nuclide i (atoms $\cdot$ g^{-1}), ϵ is the erosion rate (g $\cdot$ cm$^{-2}\cdot$ yr^{-1}), λ_i is the decay constant for nuclide i (yr^{-1}), ρ is the sample density (g $\cdot$ cm^{-3}), and Λ_{sp} is the effective attenuation length for prodution by neutron spallation.

The internal uncertainty in the exposure age $\sigma_{int}t_i$ is:

$$(\sigma_{int}t_i)^2 = \left(\frac{\partial t_i}{\partial N_i}\right)^2 \sigma N_i^2 \tag{7}$$

where σN_i is the standard error in the measured nuclide concentration and:

$$\frac{\partial t_i}{\partial N_i} = \left[P_i - N_i\left(\lambda_i + \frac{\rho\epsilon}{\Lambda_{sp}}\right)\right]^{-1} \tag{8}$$

The external uncertainty in the exposure age $\sigma_{ext}t_i$ is:

$$(\sigma_{ext}t_i)^2 = \left(\frac{\partial t_i}{\partial N_i}\right)^2 \sigma N_i^2 + \left(\frac{\partial t_i}{\partial P_i}\right)^2 \sigma P_i^2 \tag{9}$$

where

$$\frac{\partial t_i}{\partial P_i} = -N_i\left[P_i^2 - N_i P_i\left(\lambda_i + \frac{\rho\epsilon}{\Lambda_{sp}}\right)\right]^{-1} \tag{10}$$

$$\sigma P_i = \sigma P_{i,ref} * S_{thick} * S_T * S_{i,geo} \tag{11}$$

and $\sigma P_{i,ref}$ is the standard error in the reference production rate of nuclide i.

1.6 get_al_be_erosion.m

```
results = get_al_be_erosion(sample,consts,nuclide)
```

This is the main control function that carries out the erosion rate calculation. `al_be_erosion_one` and `al_be_erosion_many` call it.

The argument `sample` is a structure containing sample information. The fields are as follows:

sample.sample_name	Sample name	string
sample.lat	Latitude	double
sample.long	Longitude	double
sample.elv	elevation in meters	double
sample.pressure	pressure in hPa (optional if sample.elv is set)	double
sample.aa	Flag that indicates how to interpret the elevation value.	string
sample.thick	Sample thickness in cm	double
sample.rho	Sample density, $g \cdot cm^{-3}$	double
sample.othercorr	Shielding correction.	double
sample.N10	^{10}Be concentration, atoms $\cdot$ $g^{-1} \cdot$ yr^{-1}	double
sample.delN10	standard error of ^{10}Be concentration	double
sample.N26	^{26}Al concentration, atoms $\cdot$ $g^{-1} \cdot$ yr^{-1}	double
sample.delN26	standard error of ^{26}Al concentration	double

The argument `consts` is a structure containing the constants. It is typically the structure created by `make_al_be_consts_v12.m`, although only a subset of the fields in that structure are actually used by this function.

The argument `nuclide` tells the function which nuclide is being used; allowed values are 10 or 26. This is a numerical value, not a string.

This function returns a structure called `results` that contains the following fields:

results.main_version	Version number for this function	string
results.obj_version	Version number for the objective function used in solving for the erosion rate	string
results.Egcm2yr	Erosion rate (g $\cdot$ cm$^{-2}\cdot$ yr^{-1})	double
results.EmMyr	Erosion rate (m $\cdot$ Myr^{-1})	double
results.delE_int	Internal uncertainty (m $\cdot$ Myr^{-1})	double
results.delE_ext	External uncertainty (m $\cdot$ Myr^{-1})	double
results.P0sp	Thickness-integrated production rate due to spallation	double
results.P0muneg	Thickness-integrated production rate due to negative muon capture	double
results.P0mufast	Thickness-integrated production rate due to fast muon reactions	double
results.Nsp	Nuclide concentration attributable to production by spallation (atoms $\cdot$ g^{-1})	double
results.Nmu	Nuclide concentration attributable to production by muons (atoms $\cdot$ g^{-1})	double
results.fzero_status	Diagnostic flag indicating whether `fzero` converged or not	double
results.fzero_output	Structure containing diagnostic information supplied by `fzero`. See the MATLAB documentation of `fzero` for more information.	
results.fval	Value of objective function at solution	double
results.time	Time required to solve for the erosion rate (s)	double

This function solves the equation:

$$\int_0^\infty \left[P_{i,sp}(\epsilon t) + P_{i,\mu f}(\epsilon t) + P_{i,\mu -}(\epsilon t)\right] e^{-\lambda_i t} dt - N_i = 0 \tag{12}$$

for the erosion rate ϵ (here in g $\cdot$ cm$^{-2}\cdot$ yr^{-1}), where N_i is the measured concentration of nuclide i, and $P_{i,sp}(z)$, $P_{i,\mu f}(z)$, and $P_{i,\mu -}(z)$ are the production rates of nuclide i due to spallation, fast muon interactions, and negative muon capture, averaged over the sample thickness, as functions of depth. As this cannot be solved analytically, we use the MATLAB rootfinding algorithm `fzero` to find the zero of an objective function, `al_be_E_forward.m`, that computes the right-hand side of this equation, that is, the predicted nuclide concentration for a particular erosion rate. The actual integration is described in the documentation for `al_be_E_forward`.

As the objective function involves several numerical integrations, finding its zero can be slow. The easiest way to speed it up is to provide it with an initial guess for the solution that is close to the actual solution. We do this by first estimating the erosion rate using the simple equation of Lal (1991):

$$N_i = \frac{P_i}{\lambda_i + \frac{\epsilon}{\Lambda_{sp}}} \tag{13}$$

and then scaling this result according to a set of comparisons between the Lal erosion rate and the true erosion rate that we have obtained from calculating both at a variety of erosion rates and elevations. This usually produces an initial guess for the erosion rate that is within a few percent of the actual solution of the full equation, with the result that solving the full equation takes 1-3 seconds on our server.

The fact that the full erosion rate equation cannot be solved analytically also makes error propagation difficult. We cannot compute the derivatives of the derived erosion rate with respect to the uncertain input parameters analytically, so it requires two additional solutions of the full equation to estimate the derivative with respect to each uncertain parameter. It is time-consuming to do this using the full equation, so we use a simplified erosion rate - nuclide concentration relationship for the uncertainty analysis:

$$\frac{P_{i,sp}}{\lambda_i + \frac{\epsilon}{\Lambda_{sp}}} + \frac{P_{i,\mu}}{\lambda_i + \frac{\epsilon}{\Lambda_{\mu}}} - N_i = 0 \qquad (14)$$

where λ_i is the decay constant for nuclide i, $P_{i,sp}$ is the surface production rate due to spallation, $P_{i,\mu}$ is the surface producrion rate due to muons, Λ_{sp} is the effective attenuation length for production by spallation, and Λ_{mu} is the effective attenuation length for production by muons. Λ_{μ} varies depending on the erosion rate and the sample elevation, which is why this simplified equation cannot be used to accurately calculate the erosion rate in the first place. However, as we have already calculated $N_{i,\mu}$, the nuclide concentration attributable to production by muons, while solving the full equation above, we can calculate the value of Λ_{μ} for which the simplified equation would yield the true erosion rate by:

$$\Lambda_{\mu} = \frac{\epsilon_{true}}{\frac{P_{i,\mu}}{N_{i,\mu}} - \lambda_i} \qquad (15)$$

where ϵ_{true} is the 'true' erosion rate calculated by solving the full equation. The result is that the simplified erosion rate-nuclide concentration relationship is adequate for the uncertainty analysis. We have found that this simplified method of calculating the uncertainties yields uncertainties that, for all practical purposes, are the same as the 'true' uncertainty obtained by numerically differentiating the full erosion rate equation, for the full physically reasonable range of erosion rates and sample locations.

Thus, the simplified erosion rate equation is coded as a subfunction which can be solved quickly by `fzero`. This allows the derivatives of the erosion rate with respect to the uncertain parameters $\partial\epsilon/\partial N_i$, $\partial\epsilon/\partial P_{i,ref}$, etc. to be quickly calculated. We actually do this with a first-order centered difference scheme. We then add the uncertainties in quadrature to arrive at the final internal and external uncertainties.

1.7 make_al_be_consts_v12.m

Syntax:

```
make_al_be_consts_v12
```

This function has no arguments or outputs. It creates a structure called `al_be_consts` that contains all the constants relevant to the exposure age and erosion rate calculations, and saves it as a .mat file. All fields are numerical values except for `version`, which is a string. The fields are:

al_be_consts.version	Version number for this function	string
al_be_consts.l10	^{10}Be decay constant	yr^{-1}
al_be_consts.dell10	uncertainty in ^{10}Be decay constant	yr^{-1}
al_be_consts.l26	^{26}Al decay constant	yr^{-1}
al_be_consts.dell26	uncertainty in ^{26}Al decay constant	yr^{-1}
al_be_consts.Lsp	^{10}Be effective attenuation length for production by spallation	$g \cdot cm^{-2}$
al_be_consts.Fsp10	fraction of reference ^{10}Be production rate attributable to spallation	nondimensional
al_be_consts.Fsp26	fraction of reference ^{26}Al production rate attributable to spallation	nondimensional
al_be_consts.P10_ref	reference ^{10}Be production rate corresponding to Lal/Stone scaling scheme used in the exposure age calculation	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.delP10_ref	uncertainty in above	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.P26_ref	reference ^{26}Al production rate corresponding to Lal/Stone scaling scheme used in the exposure age calculation	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.delP26_ref	uncertainty in above	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.P10sp_ref	reference ^{10}Be production rate corresponding to Lal/Stone/Heisinger scaling scheme used in the erosion rate calculation	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.delP10sp_ref	uncertainty in above	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.P26sp_ref	reference ^{26}Al production rate corresponding to Lal/Stone/Heisinger scaling scheme used in the erosion rate calculation	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.delP26sp_ref	uncertainty in above	$atoms \cdot g^{-1} \cdot yr^{-1}$
al_be_consts.Natoms10	number density of O atoms in quartz	$atoms \cdot g^{-1}$
al_be_consts.Natoms26	number density of Si atoms in quartz	$atoms \cdot g^{-1}$
al_be_consts.k_neg10	summary yield for ^{10}Be production by negative muon capture in quartz	atoms $\cdot$ (stopped μ_-)$^{-1}$
al_be_consts.delk_neg10	uncertainty in above	atoms $\cdot$ (stopped μ_-)$^{-1}$
al_be_consts.k_neg26	summary yield for ^{26}Al production by negative muon capture in quartz	atoms $\cdot$ (stopped μ_-)$^{-1}$
al_be_consts.delk_neg26	uncertainty in above	atoms $\cdot$ (stopped μ_-)$^{-1}$
al_be_consts.sigma190_10	measured cross-section at 190 GeV for ^{10}Be production from O by fast muon reactions	$atoms \cdot cm^{-2}$
al_be_consts.delsigma190_10	uncertainty in above	$atoms \cdot cm^{-2}$
al_be_consts.sigma190_26	measured cross-section at 190 GeV for ^{26}Al production from Si by fast muon reactions	cm^{-2}
al_be_consts.delsigma190_26	uncertainty in above	cm^{-2}

1.8 makeEplot.m

Syntax:

```
filename = makeEplot(data)
```

This function creates the $\left[{}^{26}Al\right]^{*}$ / $\left[{}^{10}Be\right]^{*}$ – $\left[{}^{10}Be\right]^{*}$ plot for the exposure age results page. It makes use of numerous system calls, filenames, directories, etc. which are specific to the architecture of our web server. Thus, it is unlikely to be useful to users as a standalone function. The only reason it is documented here is so that users know what it is.

1.9 skyline_in.m

```
retstr = skyline_in(ins)
```

The MATLAB web server calls this function when the topographic shielding data input form (skyline_input.html) is submitted. It takes as input a structure containing string variables, which is supplied by the MATLAB web server. It returns a text string consisting of an output HTML document containing the results of the erosion rate calculation. The documentation for the MATLAB web server describes this process in more detail.

The input structure `ins` contains the following fields. All are string variables. These have the same names as the data-entry fields in the HTML input form.

ins.str_strike	Strike of sampled surface (degrees)
ins.str_dip	Dip of sampled surface
ins.str_az	String of space-separated azimuths
ins.str_el	String of space-separated horizon angles

Note that whole number degrees are required for all the inputs (on the basis that measurements with a greater precision than this are highly unlikely). Decimal degrees will be rejected.

The majority of this function consists of routines that check the input data to make sure it is in the expected form and is within expected bounds, and convert string variables to numerical values. After the data checking is complete, the function passes data to the function `skyline.m`, which actually carries out the erosion rate calculation and returns the result. Finally, this function assembles the output data, generates the required plots, and inserts the output data into the output HTML template.

2 Subsidiary calculation functions

2.1 al_be_E_forward.m

```
out = al_be_E_forward(x,sample,consts,target,dflag)
```

This is the objective function used by `get_al_be_erosion` to solve for the erosion rate.

The argument `x` is the erosion rate.

The argument `sample` is a structure containing abbreviated information about the sample. The fields are as follows:

sample.thickgcm2	Sample thickness ($g \cdot cm^{-2}$)	double
sample.pressure	Atmospheric pressure at sample location (hPa)	double

The argument `consts` contains nuclide-specific constants. The fields are as follows:

consts.l	decay constant	yr^{-1}
consts.Lsp	effective attenuation length for production by spallation	$g \cdot cm^{-2}$
consts.Psp0	surface production rate by spallation at sample site	$g \cdot cm^{-2}$
consts.Natoms	number density of target atoms in quartz	atoms $\cdot\ g^{-1}$
consts.k_neg	summary yield for production by negative muon capture in quartz	atoms $\cdot$ (stopped μ_-)$^{-1}$
consts.sigma190	measured cross-section at 190 GeV for production by fast muon reactions	cm^{-2}

The argument `target` is the measured nuclide concentration (atoms $\cdot\ g^{-1}$), that is, the target that the objective function is trying to match.

The argument `dflag` is a string variable telling the function what to return. If `dflag` = 'no,' the output is just the objective function value. If `dflag` = 'yes,' the output is a structure containing diagnostic information, as follows:

out.ver	Version number of this function	string
out.N_mu	Nuclide concentration in sample attributable to production by muons at the given erosion rate	atoms $\cdot\ g^{-1}$
out.N_sp	Nuclide concentration in sample attributable to production by spallation at the given erosion rate	atoms $\cdot\ g^{-1}$
out.P_fast	Thickness-integrated nuclide production rate by fast muon reactions	atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$
out.P_neg	Thickness-integrated nuclide production rate by negative muon capture	atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$
out.P_sp	Thickness-integrated nuclide production rate by spallation	atoms $\cdot\ g^{-1} \cdot\ yr^{-1}$

This function calculates the following:

$$\int_0^\infty P_{i,sp}(\epsilon t)e^{-\lambda_i t}dt + \int_0^\infty \left[P_{i,\mu f}(\epsilon t) + P_{i,\mu -}(\epsilon t)\right] e^{-\lambda_i t}dt - N_i \tag{16}$$

where ϵ is the erosion rate (the input argument `x`, here in g $\cdot$ cm$^{-2}\cdot$ yr^{-1}), N_i is the measured concentration of nuclide i (the input argument `target`), and $P_{i,sp}(z)$, $P_{i,\mu f}(z)$, and $P_{i,\mu -}(z)$ are the production rates of nuclide i due to spallation, fast muon interactions, and negative muon capture, averaged over the sample thickness, as functions of depth.

The first term of this equation, that is, the nuclide concentration in the sample attributable to production by spallation, can be integrated analytically:

$$\int_0^\infty P_{i,sp}(\epsilon t)e^{-\lambda_i t}dt = \frac{P_{i,sp}(0)\Lambda_{sp}}{\delta z\left(\lambda_i + \frac{\epsilon}{\Lambda_{sp}}\right)}\left[1 - \exp - \frac{\delta z}{\Lambda_{sp}}\right] \tag{17}$$

where $P_{i,sp}(0)$ is the surface production rate of nuclide i due to spallation (atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}), Λ_{sp} is the effective attenuation length for production by spallation (g $\cdot$ cm^{-2}), δz is the sample thickness (g $\cdot$ cm^{-2}), and λ_i is the decay constant for nuclide i (yr^{-1}).

The second term must be calculated numerically. This function uses the MATLAB numerical integration algorithm `quad` to do the integral:

$$\int_0^{t_{max}} P_{i,\mu}(\epsilon t)e^{-\lambda_i t}dt \tag{18}$$

where $P_{i,\mu} = P_{i,\mu f} + P_{i,\mu -}$ is calculated by the function `P_mu_total.m`. The upper limit of integration t_{max} is either $(2 \times 10^5)/\epsilon$ (nuclide production is insignificant below 2×10^5 g $\cdot$ cm^{-2}) or five half-lives of the relevant nuclide (a negliible number of atoms produced before this time will still be present), whichever is smaller. The integration tolerance is set at $(1 \times 10^{-4})N_i$.

2.2 angdist.m

Syntax:

```
psi = angdist(phi1,theta1,phi2,theta2)
```

This function calculates the angular distance between two points on a sphere. Given (latitude, longitude) for two points (ϕ_1, θ_1) and (ϕ_2, θ_2), the angle ψ between them is:

$$\psi = arccos\left(cos(\phi_1)cos(\phi_2)\left[cos(\theta_1)cos(\theta_2) + sin(\theta_1)sin(\theta_2)\right] + sin(\phi_1)sin(\phi_2)\right) \tag{19}$$

2.3 antatm.m

Syntax:

```
pressure = antatm(elevation)
```

This function converts elevation z in meters to atmospheric pressure p in hPa using the pressure-elevation relationship:

$$p(z) = 989.1 \exp\left[\frac{-z}{7588}\right] \tag{20}$$

The pressure-elevation relationship is derived from:

Radok, U., Allison, I., and Wendler, G., 1996. Atmospheric pressure over the interior of Antarctica. *Antarctic Science* 8, pp. 209-217.

For further discussion see:

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

2.4 d2r.m

Syntax:

```
radians = d2r(degrees)
```

This function converts angular measurements in degrees to angular measurements in radians.

2.5 P_mu_total.m

Syntax:

```
out = P_mu_total(z,h,consts,dflag)
```

This function calculates the nuclide production rate due to muons at a particular surface elevation and depth below the surface. The method is described in:

Heisinger, B., Lal, D., Jull, A.J.T., Kubik. P., Ivy-Ochs, S., Neumaier, S., Knie, K., Lazarev, V., and Nolte, E., 2002. Production of selected cosmogenic radionuclides by muons: 1. Fast muons. *Earth and Planetary Science Letters* 200, pp. 345-355. (henceforth, H2002a).

Heisinger, B., Lal, D., Jull, A.J.T., Kubik. P., Ivy-Ochs, S., Knie, K., and Nolte, E., 2002. Production of selected cosmogenic radionuclides by muons: 2. Capture of negative muons. *Earth and Planetary Science Letters* 200, pp. 357-369. (henceforth, H2002b).

The input argument `z` is depth below the surface in $g \cdot cm^{-2}$. This argument can be a vector.

The input argument `h` is the atmospheric pressure at the surface in hPa.

The input argument consts is a structure containing nuclide-specific constants. The fields are as follows:

consts.Natoms	number density of target atoms in quartz	$\text{atoms} \cdot g^{-1}$
consts.k_neg	summary yield for production by negative muon capture in quartz	$\text{atoms} \cdot (\text{stopped } \mu_{-})^{-1}$
consts.sigma190	measured cross-section at 190 GeV for production by fast muon reactions	cm^{-2}

The argument `dflag` is a string variable telling the function what to return. If `dflag` = 'no,' the output is simply a vector of production rates of the same size as the input argument `z`. If `dflag` = 'yes,' the output is a structure containing diagnostic information, as follows:

out.phi_vert_slhl	Flux of vertically traveling muons at the specified depths at sea level	muons $\cdot$ cm$^{-2}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.R_vert_slhl	Stopping rate of vertically traveling muons at the specified depths at sea level	muons $\cdot$ g$^{-1}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.phi_vert_site	Flux of vertically traveling muons at the specified depths at the site elevation	muons $\cdot$ cm$^{-2}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.R_vert_site	Stopping rate of vertically traveling muons at the specified depths at the site elevation	muons $\cdot$ g$^{-1}\cdot$ sr$^{-1}\cdot$ s^{-1}
out.phi	Total flux of muons at the specified depths at the site elevation	muons $\cdot$ cm$^{-2}\cdot$ yr^{-1}
out.R	Total stopping rate of negative muons at the specified depths at the site elevation	negative muons $\cdot$ g$^{-1}\cdot$ yr^{-1}
out.Beta	Factor describing the energy dependence of fast muon reaction cross-sections	nondimensional
out.Ebar	Mean muon energy at the specified depths	GeV
out.P_fast	Nuclide production rate by fast muon reactions at the specified depths	atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}
out.P_neg	Nuclide production rate by negative muon capture at the specified depths	atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}
out.H	Atmospheric depth between the site elevation and sea level	g $\cdot$ cm^{-2}
out.LZ	Atmospheric attenuation lengths for vertically traveling muons stopping at the specified depths	g $\cdot$ cm^{-2}

The calculation goes as follows.

1. Calculate the flux of vertically traveling muons as a function of depth at sea level and high latitude.

This is accomplished by Equations (1) and (2) in H2002a. The flux of vertically traveling muons at a depth z at sea level and high latitude $\Phi_{v,0}(z)$is:

$$\Phi_{v,0}(z) = \frac{5.401 \times 10^7}{(z + 21000)\left[(z + 1000)^{1.66} + 1.567 \times 10^5\right]} e^{-5.5\times10^{-6}z} \tag{21}$$

for depths $z < 200,000$ g $\cdot$ cm^{-2}. This is Equation (1) from H2002a, modified so that z is in g $\cdot$ cm^{-2}. For greater depths, $\Phi_v(z)$ is given by Equation (2) of H2002a, similarly modified:

$$\Phi_{v,0}(z) = 1.82 \times 10^{-6} \left[\frac{121100}{z}\right]^2 e^{-\frac{z}{121100}} + 2.84 \times 10^{-13} \tag{22}$$

The units of $\Phi_v(z)$ are muons $\cdot$ cm$^{-2}\cdot$ s$^{-1}\cdot$ sr^{-1}.

2. Calculate the stopping rate of vertically traveling muons as a function of depth at sea level and high latitude, which is equivalent to the range spectrum of vertically traveling muons at the surface.

The stopping rate of vertically traveling muons at a depth z at sea level and high latitude $R_{v,0}(z)$ is the derivative of the flux of vertically traveling muons with respect to depth. It has units of muons $\cdot$ g$^{-1}\cdot$ s^{-1}. For depths $z < 200,000$ g $\cdot$ cm^{-2},

$$R_{v,0}(z) = \frac{d}{dz}\left(\Phi_{v,0}(z)\right) = -5.401 \times 10^{7} \left[\frac{bc\frac{da}{dz} - a\left(\frac{db}{dz}c + \frac{dc}{dz}b\right)}{b^2c^2}\right] \tag{23}$$

where

$$a = e^{-5.5\times10^{-6}z} \qquad \frac{da}{dz} = -5.5 \times 10^{-6} e^{-5.5\times10^{-6}z} \tag{24}$$

$$b = (z + 21000) \qquad \frac{db}{dz} = 1 \tag{25}$$

$$c = (z + 1000)^{1.66} + 1.567 \times 10^{5} \qquad \frac{dc}{dz} = 1.66\,(z + 1000)^{0.66} \tag{26}$$

The negative sign is added because the muon flux decreases with increasing depth, so its derivative ought properly to be negative, but we would like a positive value for the stopping rate. For greater depths,

$$R_{v,0}(z) = \frac{d}{dz}\left(\Phi_{v}(z)\right) = -1.82 \times 10^{-6} \left[\frac{df}{dz}g + \frac{dg}{dz}f\right] \tag{27}$$

where

$$f = \left[\frac{121100}{z}\right]^2 \qquad \frac{df}{dz} = \frac{-2\,(121100)^2}{z^3} \tag{28}$$

$$g = e^{-\frac{z}{121100}} \qquad \frac{dg}{dz} = -\frac{e^{-\frac{z}{121100}}}{121100} \tag{29}$$

The stopping rate of vertically traveling muons as a function of depth is equivalent to the range spectrum of vertically traveling muons at the surface. That is, a muon that had a range of 1000 g $\cdot$ cm^{-2} at the surface will stop at a depth of 1000 g $\cdot$ cm^{-2}.

3. Adjust the range spectrum of vertically traveling muons to a different elevation.

First, calculate the difference in atmospheric pressure between sea level and the elevation of interest. We use the standard atmosphere approximation to convert elevation to atmospheric pressure. The atmospheric pressure in hPa as a function of elevation is:

$$P_{atm}(z) = P_{atm,0} exp\left(-\frac{0.03417}{0.0065}\left[\ln 288.15 - \ln\left(288.15 - 0.0065h\right)\right]\right) \tag{30}$$

where h is the elevation in meters and $P_{atm,0} = 1013.25$ hPa is the sea level pressure. Pressure can then be converted to the quantity of interest, that is, δz, the atmospheric depth in g $\cdot$ cm^{-2} between the site of interest and sea level, by $\delta z = 1.019716\,(P_{atm,0} - P_{atm}(z))$.

If the atmospheric depth between sea level and the site of interest is δz, then the vertically traveling muon flux at the surface as a function of muon range Z at sea level $R_{v,0}(Z)$ can be scaled to the vertically traveling muon flux at the surface as a function of muon range at the site $R_v(Z)$ by:

$$R_v(Z) = R_{v,0}(Z)e^{\frac{\delta z}{\Lambda_\mu(Z)}} \tag{31}$$

where $\Lambda_\mu(Z)$ is a range-dependent, that is, energy-dependent effective atmospheric attenuation length. These attenuation lengths are measured in studies of atmospheric muon fluxes. We follow H2002b and use a relation between muon momentum P and effective attenuation length $\Lambda_{mu}(P)$ derived from:

Boezio, M, and 33 co-authors, 2000. Measurement of the flux of atmospheric muons with the CAPRICE94 apparatus. *Physical Review D*, 62, 032007.

The relation is:

$$\Lambda_{mu}(P) = 263 + 150P \tag{32}$$

In order to use this relation, we convert muon range to momentum using tabulated values in:

Groom, D.E., Mokhov, N.V., and Striganov, S.I., 2001. Muon stopping power and range tables 10 MeV - 100 TeV. *Atomic data and nuclear data tables* 78, pp. 183-356.

Having obtained Λ_{mu} for muons stopping at the depths of interest, we can then obtain the range spectrum of vertically traveling muons at the surface at the site of interest, $R_v(Z)$, which is equivalent to the muon stopping rate as a function of depth at the site of interest $R_v(z)$, by applying Equation 31.

4. Latitudinal variability in the range spectrum.

Although the muon range spectrum is expected to change with latitude as well as elevation due to geomagnetic effects, this effect is expected to be small. We follow H2002b and ignore it.

5. Calculate the flux of vertically traveling muons at the site of interest as a function of depth.

The flux of vertically traveling muons as a function of depth at the site of interest $\phi_v(z)$ is the integral of the muon stopping rate as a function of depth at the site of interest $R_v(z)$, which we have just calculated, from infinite depth to depth z. That is, the muon flux at a particular depth is composed of all the muons which stop below that depth. Thus, the flux of vertically traveling muons as a function of depth at the site of interest $\phi_v(z)$ is:

$$\phi_v(z) = \int_z^\infty R_v(x)dx \tag{33}$$

We actually do this integral numerically, although it would be possible to obtain an analytical expression. We take the limit of integration to be 2×10^5 g $\cdot$ cm^{-2}, where the muon flux is essentially negligible for our purposes.

6. Calculate the total muon flux as a function of depth at the site of interest.

Following equations (3) - (5) in H2002a, the zenith angle dependence of the muon flux is:

$$\phi(z, \theta) = \phi_v(z)cos^{n(z)}\theta \tag{34}$$

where θ is the zenith angle and n is given by Equation (4) in H2002a, modified so that z is given in g $\cdot$ cm^{-2}:

$$n(z) = 3.21 - 0.297 \ln\left(\frac{z}{100} + 42\right) + 1.21 \times 10^{-5} z \tag{35}$$

The total muon flux at a particular depth $\phi(z)$ then consists of Equation 34 integrated over the entire upper hemisphere and has units of muons $\cdot$ cm$^{-}2\cdot$ s^{-1}. This is given by:

$$\phi(z) = \frac{2\pi}{n(z+\delta z)+1}\phi_v(z) \tag{36}$$

7. Calculate the total muon stopping rate as a function of depth at the site of interest.

The total muon stopping rate as a function of depth Rz is the derivative with respect to depth of the total muon flux as a function of depth $\phi(z)$, that is:

$$\begin{aligned}
R(z) &= \frac{d}{dz}\left(\phi(z)\right) && (37)\\
&= \frac{d}{dz}\left(\frac{2\pi}{n(z+\delta z)+1}\phi_v(z)\right) && (38)\\
&= \frac{2\pi}{n(z+\delta z)+1}\frac{d}{dz}\left(\phi_v(z)\right) - \phi_v(z)\frac{d}{dz}\left(\frac{2\pi}{n(z+\delta z)+1}\right) && (39)\\
&= \frac{2\pi}{n(z+\delta z)+1}R_v(z) - \phi_v(z)\left(-2\pi\right)\left(n(z+\delta z)+1\right)^2 \frac{d}{dz}\left(n(z+\delta z)+1\right) && (40)\\
&= \frac{2\pi}{n(z+\delta z)+1}R_v(z) - \phi_v(z)\left(-2\pi\right)\left(n(z+\delta z)+1\right)^2 \left[\frac{-0.297 \times 10^{-2}}{\frac{z+\delta z}{100}+42} + 1.21 \times 10^{-5}\right] && (41)
\end{aligned}$$

which, as we have already calculated $n(z)$, $R_v(z)$, and $\phi_v(z)$, we can calculate easily. Again, a factor of -1 is added to obtain a positive number of stopped muons.

To summarize, we have now calculated the total muon flux $\phi(z)$ (muons $\cdot$ cm$^{-2}\cdot$ s $^{-1}$) and the total stopping rate of muons $R(z)$ (muons $\cdot$ g$^{-1}\cdot$ s $^{-1}$) at our site. We convert these to muons $\cdot$ cm$^{-2}\cdot$ yr $^{-1}$ and muons $\cdot$ g$^{-1}\cdot$ yr $^{-1}$, respectively, by multiplying them by 3.154×10^7 s $\cdot$ yr^{-1}.

Finally, we compute the stopping rate of negative muons $R^{-}(z) = 0.44R(z)$ (negative muons $\cdot$ g$^{-1}\cdot$ yr $^{-1}$).

8. Calculate the nuclide production rate due to negative muon capture.

Following Equation (11) in H2002b, the production rate of nuclide i (atoms $\cdot$ g $^{-1}\cdot$ yr^{-1}) from negative muon capture $P_{i,\mu-}(z)$ is:

$$P_{i,\mu-}(z) = R^{-}(z) f_{i,C} f_{i,D} f_i^{*} \tag{42}$$

where

$$f_{10,C} = 0.704 \qquad f_{26,C} = 0.296 \tag{43}$$
$$f_{10,D} = 0.1828 \qquad f_{26,D} = 0.6559 \tag{44}$$
$$f^*_{10} = 0.0043 \qquad f^*_{26} = 0.022 \tag{45}$$

9. Calculate the nuclide production rate due to fast muon reactions.

Following Equation (14) in H2002a, the production rate of nuclide i (atoms · g $^{-1}$· yr^{-1}) from fast muon interactions $P_{i,\mu fast}(z)$ is:

$$P_{i,\mu fast}(z) = \phi(z)\sigma_{0,i}\beta(z)\left(\bar{E}(z)\right)^{\alpha} N_{t,i} \tag{46}$$

where $\alpha = 0.75$.

$\sigma_{0,i}$ is the nominal zero-energy muon interaction cross-section for the reaction responsible for producing nuclide i. Here is has units of cm^{-2}. The muon interaction cross-section for a particular reaction is thought to depend on the muon energy as follows:

$$\sigma_i(E) = \sigma_{0,i}E^{\alpha} \tag{47}$$

$\sigma_{0,i}$ is determined from the measured cross-sections $\sigma_i(E)$ at 190 GeV energy in Table 1 of H2002a using this equation. These values for ^{10}Be and ^{26}Al are:

$$\sigma_{0,10} = \frac{0.094 \times 10^{-27}}{190^{\alpha}} \qquad \sigma_{0,26} = \frac{1.41 \times 10^{-27}}{190^{\alpha}} \tag{48}$$

$N_{t,i}$ is the number density of atoms of the target element (atoms · g^{-1}). The values for O and Si relevant to ^{10}Be and ^{26}Al production respectively are:

$$N_{t,10} = 2.006 \times 10^{22} \qquad N_{t,26} = 1.003 \times 10^{22} \tag{49}$$

where β is a function of depth and is approximated by Equation (16) of H2002a:

$$\beta(z) = 0.846 = 0.015 \ln\left(\frac{z}{100} + 1\right) + 0.003139 \left[\ln\left(\frac{z}{100}\right)\right] \tag{50}$$

and $\bar{E}(z)$ is the mean muon energy at depth z and is given by Equation (11) of H2002a:

$$\bar{E}(z) = 7.6 + 321.7\left(1 - e^{-8.059\times10^{-6}z}\right) + 50.7(1 - e^{-5.05\times10^{-7}z}) \tag{51}$$

2.6 r2d.m

Syntax:

```
degrees = r2d(radians)
```

This function converts angular measurements in radians to angular measurements in degrees.

2.7 skyline.m

Syntax:

```
[out,data,ver] = skyline(az,el,strike,dip)
```

This function calculates the topographic shielding correction to the cosmic-ray flux at a site whose horizon is obstructed either by a dipping surface or by the surrounding topography.

The input arguments `strike` and `dip` are the strike and dip of the sampled surface in degrees.

The input arguments `az` and `el` are vectors containing the azimuths (degrees; $0 < \mathrm{az} < 360$; north = 0) and horizon angles (degrees; $0 < \mathrm{el} < 360$; zenith = 90) of points on the horizon.

`out` is the shielding correction for the sample (nondimensional; the ratio of the nuclide production rate at the shielded site to the nuclide production rate at an unshielded site at the same location).

`data` is a vector containing the interpolated horizon angle visible to the sample, in degrees, for 1-degree increments of azimuth. It can be used to plot the horizon, e.g., `plot(0:360,data)`

`ver` is a string variable containing the version number of the function.

The calculation goes as folllows:

First, we divide the range of azimuths $0 < \phi < 2\pi$ into 1-degree increments.

Second, we calculate the horizon angle θ of the dipping surface as a function of azimuth:

$$\theta = \arctan\left[\tan\theta_d \cos\left(\phi - \phi_s\right)\right] \tag{52}$$

where θ_d is the dip and ϕ_d is the strike of the surface.

Third, we interpolate the supplied horizon points to give the horizon angle of the topography as a function of azimuth.

Fourth, at each increment of azimuth, we take the higher of the horizon angle due to the dipping surface or the horizon angle due to the surrounding topography to be the horizon angle visible to the sample. This results in a vector $\theta(\phi)$ which defines the visible horizon angle θ at each increment of azimuth ϕ.

Finally, the fraction of the cosmic-ray flux $f(\phi)$ that lies below the horizon angle in the increment of azimuth at ϕ is:

$$f(\phi) = \frac{\delta\phi}{2\pi}\left(\sin\theta\right)^{3.3} \tag{53}$$

where $\delta\phi$ is the width of the azimuth increment in radians ($\pi/180$). The total fraction of the cosmic-ray flux that lies below the horizon f is the sum of the values of $f(\phi)$ for each increment of ϕ. The topographic shielding correction is then $(1 - f)$.

2.8 stdatm.m

Syntax:

```
pressure = stdatm(elevation)
```

This function converts elevation $z$in meters to atmospheric pressure p in hPa using the pressure-elevation relationship in the ICAO Standard Atmosphere:

$$p(z) = p_s exp\Big\{ -\frac{gM}{R\xi} \left[\ln T_s - \ln\left(T_s - \xi z\right)\right] \Big\} \tag{54}$$

where sea level pressure $P_s = 1013.25$ hPa; sea level temperature $T_s = 288.15$ K; and the adiabatic lapse rate $\xi = 0.0065$ K $\cdot$ m^{-1}. M is the molar weight of air, g the acceleration due to gravity, and R the gas constant, giving $gM/R =$ 0.03417 K $\cdot$ m^{-1}.

For further discussion see:

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

2.9 stone2000.m

Syntax:

```
scalingfactor = stone2000(latitude, pressure, fsp)
```

Calculates the geographic scaling factor for cosmogenic-nuclide production with varying latitude and altitude according to the scheme in:

Stone, J., 2000. Air pressure and cosmogenic isotope production. *Journal of Geophysical Research* 106, pp. 23,753-23,759.

The input arguments are `latitude`, the site latitude (decimal degrees), `pressure`, the atmospheric pressure at the site (hPa), and `fsp` the fraction of production due to neutron spallation (nondimensional). Accepts vector arguments. The argument `fsp`, if omitted, defaults to 0.978, which is the correct value for ^{10}BeṪhe corresponding value for ^{26}Al is 0.974. Note that using `fsp` = 0.844 (^{10}Be) and `fsp` = 0.844 (^{26}Al) will closely reproduce the scaling factors given in Lal (1991) as long as the standard atmosphere is used. Note also that this function can be made to yield the scaling factor for spallation only by specifying `fsp` = 1 and the scaling factor for production by muons only by specifying `fsp` = 0.

The geographic production rate scaling factor S_{geo} is:

$$S_{geo}(t) = f_{sp}S_{sp} + (1 - f_{sp})S_{\mu} \tag{55}$$

where f_{sp} is the fraction of production due to spallation (input argument `fsp`), S_{sp} is the scaling factor for production by neutron spallation:

$$S_{sp}(p) = a + b\exp\frac{-p}{150} + cp + dp^2 + ep^3 \tag{56}$$

and S_{μ} is the scaling factor for production by muons:

$$S_{i,\mu}(p) = M_{sl}\exp\left(\frac{1013.25 - p}{242}\right) \tag{57}$$

Where the constants $a...e$ and M_{sl} depend on the latitude and are defined for certain index latitudes:

Latitude	a	b	c	d	e	M_{sl}
0°	31.8518	250.3193	-0.083393	7.4260e-5	-2.2397e-8	0.587
10°	34.3699	258.4759	-0.089807	7.9457e-5	-2.3697e-8	0.600
20°	40.3153	308.9894	-0.106248	9.4508e-5	-2.8234e-8	0.678
30°	42.0983	512.6857	-0.120551	1.1752e-5	-3.8809e-8	0.833
40°	56.7733	649.1343	-0.160859	1.5463e-5	-5.0330e-8	0.933
50°	69.0720	832.4566	-0.199252	1.9391 e-5	-6.3653e-8	1.000
$>$ 60°	71.8733	863.1927	-0.207069	2.0127e-5	-6.6043e-8	1.000

This table duplicates Table 1 in Stone (2000).

This function actually calculates S_{sp} and S_{mu} for index latitudes that bound the actual latitude of the site, and then determines the value at the site by linear interpolation. The exact method of interpolation used is relatively unimportant, but we have chosen linear interpolation to avoid the problem of under- and over-shooting between index latitudes due to the polynomial form of the Lal equations. Basically, linear interpolation guarantees that the scaling factor at the site is not outside the range of the scaling factors at the bounding index latitudes.

2.10 thickness.m

Syntax:

```
Sthick = thickness(zmax, Lambdasp, rho)
```

Calculates the thickness correction factor for nuclide production by spallation. The arguments are `zmax`, the sample thickness in cm; `Lambdasp`, the effective attenuation length for production by neutron spallation in atoms $\cdot$ g$^{-1}\cdot$ yr^{-1}; and `rho`, the sample density in g $\cdot$ cm^{-3}. Accepts vector inputs.

Calculating the thickness correction factor S_{thick} begins by assuming that the production rate decreases with depth according to a single exponential function:

$$P(z) = P(0)\exp(\frac{\rho z}{\Lambda_{sp}}) \tag{58}$$

where $P(0)$ is the surface production rate and z is depth below the surface (cm). The thickness scaling factor S_{thick} is then:

$$S_{thick} = \frac{1}{z_{max}} \int_0^{z_{max}} P(z)dz = \frac{\Lambda_{sp}}{\rho z_{max}} \left[1 - \exp(\frac{-\rho z_{max}}{\Lambda_{sp}})\right] \tag{59}$$

where Λ_{sp} is the effective attenuation length for production by spallation (input argument `Lambdasp`) and ρ is the sample density (input argument `rho`).

3 Plotting functions

3.1 ellipse.m

Syntax:

```
[x,y] = ellipse(N10,delN10,N26,delN26,0,plotString)
[h1] = ellipse(N10,delN10,N26,delN26,1,plotString)
[h1,h2] = ellipse(N10,delN10,N26,delN26,2,plotString)
```

Plots an uncertainty ellipse on the [^{10}Be]* – [^{26}Al]* / [^{10}Be]* diagram (the ‘Lal-Klein-Nishiizumi Al-Be diagram’).

The arguments `N10`, `delN10`, `N26`, and `delN26` are the ^{10}Be and ^{26}Al concentrations and their uncertainties (atoms · g^{-1}).

The third argument can be 0, in which case the function returns the x and y coordinates of the $1-\sigma$ uncertainty ellipse; 1, in which case the function plots the 1σ uncertainty ellipse and returns its handle; or 2, in which case the function plots 1σ and 2σ uncertainty ellipses and returns their handles. The default is 1.

The argument `plotString` is an optional string telling the function what line type and color to use to draw the ellipse. See the documentation for the MATLAB function `plot` for more information.

What this function actually does is calculate the joint probability on a mesh in (x, y):

$$p(x,y) = x \exp\left(-0.5\left[\frac{xy - N_{26}}{\sigma N_{26}}\right]^2 + \left[\frac{x - N_{10}}{\sigma N_{10}}\right]^2\right) \tag{60}$$

It then creates a normalized cumulative distribution for p to determine the values of p that account for 68% and 95% of the total probability, and draws contours of the 2-d PDF at those values.