CRONUS-Earth $^{26}$Al-$^{10}$Be exposure age calculator – Version 1

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Contents

1 Overall goals and disclaimer 3

2 How the calculator works 3
  2.1 Basic structure ................................................................. 4
  2.2 List of inputs ................................................................. 4
  2.3 List of outputs ............................................................... 6
  2.4 List of constants ............................................................ 7
    2.4.1 Reference production rates ........................................ 8
    2.4.2 Physical constants .................................................. 8
    2.4.3 Paleomagnetic records ............................................. 9

3 Descriptions of individual MATLAB functions 9
  3.1 Direct access to MATLAB scripts: .................................... 9
  3.2 Wrapper script: al_be_age_one.m ..................................... 9
  3.3 Main age calculation: get_al_be_age.m .............................. 10
  3.4 Objective function for paleomagnetic corrections: al_be_forward.m .................................................... 12
  3.5 Subsidiary functions .................................................... 14
    3.5.1 Angular distance measurement: angdist.m ...................... 14
    3.5.2 Elevation/pressure relationships: stdatm.m and antatm.m 14
    3.5.3 Angle conversions: r2d.m and d2r.m ............................ 15
    3.5.4 Geographic scaling factor: stone2000.m ..................... 15
    3.5.5 Thickness scaling factor: thickness.m ...................... 15
    3.5.6 Geometric shielding: skyline.m .................................. 15

4 References 15
1 Overall goals and disclaimer

The goal of this first version of the CRONUS-Earth online exposure age calculator is to codify standard practices for calculating exposure ages from cosmogenic nuclide concentrations in a central and easily accessible location. The goal of this first section of the documentation is to transmit boring but necessary information about what these calculators are and are not intended to accomplish – anyone who uses these calculators should do their best to read the following.

We are motivated to develop this exposure-age calculator in the first place by the fact that the number of applications of cosmogenic-nuclide exposure dating, as well as the number of papers published on the subject, has been rapidly growing. Also, these studies are no longer being carried out exclusively by exposure-dating specialists, but by geologists and paleoclimatologists who wish to use exposure ages as part of a broader study. As the technique of exposure dating is still under development, a variety of data-reduction methods, reference nuclide production rates, and production rate scaling schemes exist in the literature. Many of these schemes are at least in part inconsistent with each other, and yield different exposure ages for the same measurements of nuclide concentrations. The effect of this has been that published exposure-dating studies lack a common basis for exposure-age calculations. Even without worrying about the accuracy relative to the true calendar year time scale of any of the calculation methods, this proliferation of inconsistent results makes it difficult for nonspecialists even to compare the results of any two exposure-dating studies. Thus, our goal is to provide a standard method of exposure-age calculation that will allow anyone to easily calculate an exposure age or compare previously published cosmogenic-nuclide measurements in a consistent fashion.

The point of this is that these calculators are intended to give a self-consistent result that reflects standard exposure-dating practices. We can’t at this time guarantee that they provide the ‘right’ answer, that is, that they will always yield the correct calendar age for samples of all locations and ages. There are still many systematic uncertainties in nuclide production rates and scaling factors, and the ones we have chosen to use here may not prove to be the most accurate when more calibration data are available in future. We have chosen production rates and scaling factors that are relatively straightforward to understand and use, are consistent with the calibration measurements that are available, and are as consistent as possible with the majority of common usage in the existing literature. The purpose of the CRONUS-Earth project in general is to improve these production rate scaling schemes in two ways, first by better understanding the physics of cosmogenic-nuclide production, and, second, by collecting a larger calibration data set to better evaluate the various production rates and scaling schemes that have been proposed in the past and will no doubt be proposed in the future. In future, we will have a better basis for choosing the most accurate of the available production rates and scaling factors, and at that time we will issue a new version of these calculators to take account of these improvements.

One critically important part of this effort, that of providing a self-consistent means of comparing exposure ages from different studies, is that it cannot succeed unless everyone who publishes cosmogenic-nuclide exposure ages also reports all the information needed to calculate the ages. In most cases this means the location and elevation of the sample site, the density, thickness and shielding geometry of the sample, any independent information about the erosion rate, and the measured nuclide concentrations and corresponding analytical uncertainties. In other cases (e.g., complicated geometric corrections, unusual exposure histories), there are additional data that are also needed. If these data don’t appear in full in a paper or associated data repository, then the exposure ages cannot be recalculated either for comparison with other data sets or to reflect future improvements in the accuracy of production rates or scaling factors, and the study will be effectively useless to future researchers.

2 How the calculator works

The calculator uses the MATLAB web server, a system which passes inputs from HTML input forms that can be read with any web browser to MATLAB, which is a high-level programming language interpreter designed for scientific computing. It then returns the results of the calculation in another web page. The system runs on a LINUX server at the
University of Washington. We chose to build a centralized Web server rather than distributing a compiled application for two reasons: first, because the web-based input and output is platform-independent, and second, because if we maintain one central copy of the software rather than distributing many copies to users, it is easier to ensure that everyone is using the same, up-to-date, version and has access to new features as soon as they are available. We chose to use the MATLAB language because it is commonly used by geoscientists, it is easier to learn and more easily extensible than lower-level languages such as Java or C, and because many of the researchers involved in CRONUS have already written a wide variety of data-reduction software in MATLAB.

2.1 Basic structure

The calculation follows the following basic structure. The data input form, that is, the HTML document at the following address:

http://hess.ess.washington.edu/math/AlBe/al_be_input.php

is an HTML form with text boxes for entering data about a sample. When submitted, this form passes the inputs to the MATLAB web server. The MATLAB web server then calls a MATLAB wrapper script called al_be_age_one.m and passes it the input variables from the form.

The wrapper script checks to make sure the input data have the correct form and are within expected bounds, loads a data file containing the required constants (production rates, decay constants, paleomagnetic intensity curves, etc.), and separates the input data into the separate (but overlapping) data sets that pertain to the $^{10}$Be and $^{26}$Al measurements, if both are present. It then passes the data along again, to the main MATLAB script that is responsible for the bulk of the calculations, called get_al_be_age.m.

This main script makes use of a series of subsidiary functions to compute site-specific scaling factors and exposure ages, and returns the results to the wrapper script.

The wrapper script repackages the output data in the form of an output HTML document, and returns this HTML document to the MATLAB web server.

Finally, the MATLAB web server returns the output document to the client browser. Figure 1 gives an idea of the information flow, and we describe the chain of events in each of these separate functions in detail in the following sections.

2.2 List of inputs

Table 1 describes the list of inputs needed to calculate an exposure age.

Comments about choices of inputs. In order to ensure the highest degree of consistency between exposure ages calculated using this system, one ought to set up the required inputs for this calculation such that only direct measurements are used as inputs to the calculation, and all derived quantities are produced internally by the calculator. We’ve violated this rule in two places: by requesting a derived shielding factor, rather than the actual measurements of site and horizon geometry; and by requesting nuclide concentrations in atoms $\cdot$ g$^{-1}$, rather than the direct measurements of isotope ratio, sample weight, Be carrier weight, etc.
Table 1: Data needed to calculate a $^{10}$Be or $^{26}$Al exposure age

<table>
<thead>
<tr>
<th>What?</th>
<th>Units</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample name</td>
<td>Text</td>
<td>Sample names are limited to 24 characters, which is more than one can fit on the side of an AMS cathode anyway. For obvious reasons, sample names should not contain spaces or other characters which might be interpreted as delimiters, e.g. commas, semicolons, backslashes, etc.</td>
</tr>
<tr>
<td>Latitude</td>
<td>Decimal degrees</td>
<td>South latitudes should be given as negative numbers.</td>
</tr>
<tr>
<td>Longitude</td>
<td>Decimal degrees</td>
<td>West longitudes should be given as negative numbers, that is, longitudes outside the range -180 to 180 won’t be recognized.</td>
</tr>
<tr>
<td>Elevation (or atmospheric pressure)</td>
<td>Meters (or hPa)</td>
<td>Sample elevation can be specified as either meters above sea level or as mean atmospheric pressure at the site. The input forms have a menu to indicate which one is provided and, if elevation is provided, which pressure-height relationship should be used to convert it to pressure.</td>
</tr>
<tr>
<td>Thickness</td>
<td>cm</td>
<td>Sample thickness.</td>
</tr>
<tr>
<td>Density</td>
<td>g · cm$^{-3}$</td>
<td>Density of the sample, preferably directly measured. For an outline of how to measure rock density, see <a href="http://depts.washington.edu/cosmolab/chem.html">http://depts.washington.edu/cosmolab/chem.html</a>.</td>
</tr>
<tr>
<td>Shielding correction</td>
<td>Number between 0 and 1</td>
<td>The shielding correction is intended to account for obstruction of the cosmic-ray flux by topography. It’s the ratio of the production rate at the obstructed site to the production rate at a site at the same location and elevation, but with a flat surface and a clear horizon. We provide a separate page for calculating this value, which is described below in section 3.5.6.</td>
</tr>
<tr>
<td>Erosion rate</td>
<td>cm · yr$^{-1}$</td>
<td>The erosion rate of the sample surface inferred from independent evidence, to be taken into account when computing the exposure age.</td>
</tr>
<tr>
<td>Nuclide concentrations</td>
<td>atoms · g$^{-1}$</td>
<td>$^{10}$Be and $^{26}$Al concentrations in quartz in the sample. For the time being, nuclide concentrations must be normalized to the LLNL standards, which is a potential source of confusion – see discussion in Section 2.2.</td>
</tr>
<tr>
<td>Uncertainties in nuclide concentrations</td>
<td>atoms · g$^{-1}$</td>
<td>1-standard error analytical uncertainties in the measured nuclide concentrations. Should account for all sources of analytical error, e.g. AMS uncertainty, blank uncertainty, etc.</td>
</tr>
</tbody>
</table>
With regard to the shielding calculation, we’ve chosen to require a shielding factor rather than a series of measurements specifying the site and horizon geometry simply because there is no standard way of collecting this information. This is a potentially significant long-term shortcoming of our approach in light of our goal of making all calculations fully reproducible in future – if a publication reports only the derived shielding factor, and not the actual shielding geometry, then the actual geometry cannot be recovered to take advantage of future improvements in the shielding calculation. However, at this point various researchers use different methods for measuring the shielding geometry, and we’ve decided not to enforce a particular one in the calculator. We do provide a separate calculator for computing the shielding factor according to one commonly used method. This calculator is linked from the main input page of the exposure dating calculator, and we describe it below in Section 3.5.6.

With regard to the nuclide concentrations, again, in principle, we should have only the actual laboratory/AMS measurements as input, and determine the nuclide concentrations online. However, there are two problems with this. First, transforming the raw isotope ratio measurement into a derived nuclide concentration is somewhat interpretive, particularly with regard to the choice of a blank value; and second, various AMS facilities use different isotope ratio standards. Thus, at present we’re asking the user to get from their isotope ratio measurement to a nuclide concentration, normalized to the LLNL standards, offline. It’s not clear what the best way to deal with this in future is. As a stopgap measure, we plan to provide a separate calculator page to help users get from raw AMS and lab measurements to nuclide concentrations and uncertainties. Later, we will probably need to have separate input pages tailored to results from particular AMS facilities.

2.3 List of outputs

This first version of the calculator returns the following information:

1. Exposure age and uncertainty, for each nuclide measured

   (a) Simple age \( t_{\text{simple}} \). The exposure age of the sample calculated according to Equation 5. This assumes a fixed production rate through time, steady erosion at a specified erosion rate, and a production rate decreasing exponentially with depth with a single attenuation length.

   (b) Paleomagnetically corrected age \( t_{\text{corrected}} \). The exposure age of the sample taking account of the time variation in nuclide production rates due to past changes in the position of the magnetic pole and the magnetic field strength. This calculation is described below in sections 3.3 and 3.4.

   (c) Internal uncertainty. 1-standard-error uncertainty in the exposure age contributed by analytical error in the nuclide concentration measurement. This uncertainty should be used when comparing exposure ages...
of similar age and similar geographic location to each other, for example, if asking whether the observed scatter in exposure ages from boulders on a particular moraine can be attributed to analytical uncertainty alone, or must also reflect differences in the exposure history of the boulders. Both the simple age and the paleomagnetically corrected age have the same internal uncertainty.

(d) **External uncertainty.** 1-standard-error uncertainty in the exposure age taking account of both the analytical uncertainty in the nuclide concentration and the uncertainty in the reference nuclide production rate. This uncertainty should be used when comparing exposure ages to dates obtained with another method, for example calibrated radiocarbon ages or calendar ages from layer counting in ice cores. The simple age and the paleomagnetically corrected age are considered to have the same external uncertainty for the present.

2. $^{26}$Al/$^{10}$Be ratio and uncertainty therein. If the sample has both $^{26}$Al and $^{10}$Be measurements, we report their ratio, the uncertainty in the ratio, and plot the sample on the familiar $^{10}$Be – $^{26}$Al/$^{10}$Be diagram. We use the variant of the diagram in which both nuclide concentrations are normalized to their respective production rates.

3. Derived scaling factors. The production rate at the sample site is the reference production rate at sea level and high latitude, multiplied by three scaling factors: one that accounts for the sample thickness, one that accounts for the location and elevation of the sample, and one that accounts for topographic shielding of the sample site. The first two of these are computed internally, and the third is user-supplied as we discuss above. We report them all so that the user can recover the production rate at the sample site.

(a) **Thickness scaling factor $S_{\text{thick}}$.** Accounts for the fact that a thick sample sees a lower production rate than an ideal, infinitely-thin, sample from the rock surface. This is computed by integrating the depth-production rate function over the sample thickness. Details are below in Section 3.5.5.

(b) **Topographic shielding factor $S_{T}$.** This is supplied by the user, as we discuss above. We report it again in the results for completeness.

(c) **Simple geographic scaling factor $S_{\text{geo}}$.** This scaling factor accounts for the variation in production rates with geographic location and altitude, on the basis that the production rate does not change through time. Under this assumption, the nuclide production rate at the sample site is:

$$P = P_{\text{ref}} \cdot S_{\text{thick}} \cdot S_{T} \cdot S_{\text{geo}}$$

where $P_{\text{ref}}$ is the reference production rate calculated on the basis of time-invariant production. Inserting this resulting site production rate into Equation 5 below yields the simple exposure age. This scaling factor is derived from those of Lal (1991), using the formula in Stone (2000).

(d) **Time-integrated production rate scaling factor $S_{\text{GEO}}$.** This scaling factor takes into account the past variation in production rates due to paleomagnetic field variations. This scaling factor is derived from the paleomagnetically-corrected exposure age, not used in computing it. It’s mainly provided as a convenience: one can compute an ‘effective site production rate’ that takes account of paleomagnetic variation by:

$$P_{\text{eff}} = P_{\text{ref, pmag}} \cdot S_{\text{thick}} \cdot S_{T} \cdot S_{\text{GEO}}$$

where $P_{\text{ref, pmag}}$ is the reference production rate calculated on the basis of time-varying production. Using $P_{\text{eff}}$ in Equation 5 then yields the paleomagnetically corrected exposure age.

4. **Diagnostics.** The final section of the results includes some diagnostic values intended to make sure that the root-finding routine involved in finding the paleomagnetically corrected age runs to completion and provides reasonable results. This is useful for debugging and development, but we will probably remove it in future versions.

### 2.4 List of constants

The reference production rates, physical constants, and paleomagnetic records needed for the calculation are stored in a data file which is provided as input to all the internal functions in the calculator. The MATLAB function make_al_be_consts_v1.m generates this file.
2.4.1 Reference production rates

We estimated reference $^{10}$Be and $^{26}$Al production rates by following John Stone’s calculations which resulted in the commonly accepted $^{10}$Be production rate of 5.1 atoms · g$^{-1}$ · yr$^{-1}$. Although never published in their entirety, these calculations were described in detail at AMS-8 and the Lamont meeting, and are then referred to in Gosse and Stone (EOS article, date??) and Stone (2000). Basically they consist of assembling $^{10}$Be and $^{26}$Al measurements from seven geological calibration sites and two water-target experiments, scaling these measurements to sea level and high latitude using the scaling factors of Stone (2000), and choosing a reference production rate that minimizes the chi-squared misfit to all of the measurements.

The geological calibration sites are the Sierra Nevada sites of Nishiizumi (1989), the Titcomb Basin moraine of Gosse (??), the Kofels landslide of Kubik et al. (??), the An Teallach site of Stone (??), the New Jersey site of Clark et al. (1995), and unpublished measurements from Breque, Peru (Farber) and the Bonneville shoreline (Lifton). The water target experiments are those of Nishiizumi (??) at Mt. Evans and Scripps. In a future version of this document we will tabulate these existing measurements and encourage the responsible parties to check their accuracy and update them to reflect new age constraints.

The important point for the present purpose is that as we use two scaling schemes in the calculator, the Lal/Stone scaling scheme which assumes time-invariant production and a similar scaling scheme which takes account of paleomagnetic variation, we must derive two separate reference production rates for each nuclide by applying both scaling schemes to the calibration measurements separately. Thus, different reference production rates are used for calculating the simple age and the paleomagnetically-corrected age.

One disadvantage (of many) in the present calibration data set is that there are few $^{26}$Al measurements. On the premise that the $^{26}$Al/$^{10}$Be production ratio is probably better established than the actual $^{26}$Al production rate, we define the $^{26}$Al production rate to be 6.1 times the $^{10}$Be production rate.

The production rates that result from this exercise and are in use in the calculator are as follows: for the simple age calculation, 5.08±0.3 and 31.0±1.8, and for the paleomagnetically corrected age calculation, 5.23±0.3 and 31.9±1.8 atoms · g$^{-1}$ · yr$^{-1}$, for $^{10}$Be and $^{26}$Al respectively.

2.4.2 Physical constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{10}$</td>
<td>$^{10}$Be decay constant</td>
<td>$4.59 \pm 0.18 \times 10^{-7}$ yr$^{-1}$</td>
<td>ENSDF summary value</td>
</tr>
<tr>
<td>$\lambda_{26}$</td>
<td>$^{26}$Al decay constant</td>
<td>$9.78 \pm 0.25 \times 10^{-7}$ yr$^{-1}$</td>
<td>Weighted mean of values cited in ENSDF</td>
</tr>
<tr>
<td>$\Lambda_{sp}$</td>
<td>Effective attenuation length for spallogenic production in rock</td>
<td>160 g · cm$^{-2}$</td>
<td>Generally accepted value. See discussion in Gosse and Phillips (2001)</td>
</tr>
<tr>
<td>$\Lambda_{sp}$</td>
<td>Approximate effective attenuation length for production by muons in rock</td>
<td>1500 g · cm$^{-2}$</td>
<td>Middleton and Klein (1987)</td>
</tr>
<tr>
<td>$f_{sp,10}$</td>
<td>Fraction of surface $^{10}$Be production attributable to spallation</td>
<td>0.978</td>
<td>Stone (2000)</td>
</tr>
<tr>
<td>$f_{sp,26}$</td>
<td>Fraction of surface $^{26}$Al production attributable to spallation</td>
<td>0.974</td>
<td>Stone (2000)</td>
</tr>
</tbody>
</table>
2.4.3 Paleomagnetic records

Calculating a paleomagnetically corrected exposure age requires two records of past magnetic field changes: a record of past pole positions, and a record of past changes in magnetic field strength. Pigati and Lifton (2004) have compiled such records from a variety of sources, and we directly adopt their compilations here.

3 Descriptions of individual MATLAB functions

3.1 Direct access to MATLAB scripts:

Links to all of the MATLAB functions that are needed for the calculator to work are here:

http://hess.ess.washington.edu/math/AlBe/functionlist.html

They consist of the following:

1. Main control functions:
   (a) Wrapper script: al_be_age_one.m
   (b) Main age calculation: get_al_be_age.m
   (c) Objective function for paleomagnetic corrections: al_be_forward.m

2. Subsidiary functions:
   (a) Angular distance measurement: angdist.m
   (b) Elevation/air pressure relationships: stdatm.m and antatm.m
   (c) Geographic scaling factor: stone2000.m
   (d) Thickness scaling factor: thickness.m
   (e) Topographic shielding factor: skyline.m
   (f) Angle conversions: d2r.m and r2d.m
   (g) Function to generate data file of constants: make_al_beconsts_v1.m

The following sections contain detailed descriptions of all of these functions.

3.2 Wrapper script: al_be_age_one.m

The point of the wrapper script is to interact directly with the MATLAB web server. It receives a collection of string variables, containing whatever the user typed into the input form, from the server. It then takes the following actions:

1. First, check the variables to make sure all the needed variables exist, that they are of the appropriate type and size, and that they are within the required bounds. If the variables don’t pass any one of a variety of tests, attempt to return an error message telling the user what the problem is.
2. If all the input variables pass these checks, assemble a structure array containing all the sample information. The structure has fields that correspond to the input data discussed above in section 2.2.

3. Call the main calculation function, get_al_be_age.m, either once or twice, depending on whether one or both nuclides were measured. This produces one or two structures containing results of the exposure age calculation.

4. If both nuclides were measured, calculate the $^{26}\text{Al}/^{10}\text{Be}$ ratio and uncertainty. If the $^{26}\text{Al}/^{10}\text{Be}$ ratio is denoted by $R_{26/10}$, its 1$\sigma$ uncertainty is $\sigma R_{26/10}$. Assuming linear, uncorrelated uncertainties:

$$
(\sigma R_{26/10})^2 = \left(\frac{\sigma N_{26}}{N_{10}}\right)^2 + \sigma N_{10}^2 \left(\frac{-N_{26}}{N_{10}^2}\right)^2
$$

where $N_i$ is the concentration of nuclide $i$ and $\sigma N_i$ is its 1$\sigma$ analytical uncertainty.

5. Finally, assemble the data into a series of string variables, insert them into the output HTML template, and return the finished output document to the MATLAB web server.

The function is here:

http://hess.ess.washington.edu/math/AlBe/al_be_age_one.m

### 3.3 Main age calculation: get_al_be_age.m

This function takes three input parameters:

1. a structure array containing the sample information that is assembled by al_be_age_one.m
2. a structure array containing the needed production rates, physical constants, and paleomagnetic variation records (the function make_al_be_consts_v1.m produces this structure array and saves it as a .mat file which can be loaded as needed)
3. a flag indicating the nuclide of interest, that is, either 10 or 26.

It then takes the following steps:

1. First, select appropriate values for nuclide concentrations and constants, depending on which nuclide is selected.
2. Calculate the thickness scaling factor $S_{thick}$ by calling the function thickness.m (see description below in section 3.5.5).
3. Determine whether or not the elevation or the atmospheric pressure was submitted. If elevation, calculate the atmospheric pressure according to either the ICAO standard atmosphere or the Antarctic atmosphere, using the functions stdatm.m or antatm.m respectively (see section 3.5.2 below). If the atmospheric pressure was supplied, do nothing.
4. Calculate the simple geographic scaling factor $S_{geo}$ as a function of latitude and atmospheric pressure according to the polynomials of Lal (1991) as recast in atmospheric pressure by Stone (2000). This just consists of a call to the function stone2000.m.
5. Calculate the site production rate $P_i$ as follows:

$$P_i = P_{i,\text{ref}} \times S_{\text{thick}} \times S_T \times S_{i,\text{geo}}$$  \(4\)

where $P_{i,\text{ref}}$ is the reference production rate for nuclide $i$ at sea level and high latitude for time-invariant production, $S_{\text{thick}}$ is the thickness scaling factor, $S_T$ is the shielding correction, and $S_{i,\text{geo}}$ is the geographic scaling factor for nuclide $i$.

6. The simple exposure age $t_{\text{simple}}$ for nuclide $i$ is then:

$$t_{\text{simple}} = \frac{1}{\lambda_i + \frac{\rho E}{\Lambda_{\text{sp}}}} \ln \left[ \frac{N_i}{P_i} \left( \lambda_i + \frac{\rho E}{\Lambda_{\text{sp}}} \right) \right]$$  \(5\)

7. Approximate the internal uncertainty of the exposure age $\sigma_{\text{int} t}$, which only accounts for measurement error in the nuclide concentration, by assuming linear, uncorrelated uncertainties and adding in quadrature. This value is for all practical purposes the same for both the simple and paleomagnetically-corrected exposure ages, so we only calculate it once:

$$\left(\sigma_{\text{int} t}\right)^2 = \frac{\partial t_{\text{simple}}}{\partial N_i}^2 \sigma_{N_i}^2$$  \(6\)

where

$$\frac{\partial t_{\text{simple}}}{\partial N_i} = \left[ P_i - N_i \left( \lambda_i + \frac{\rho E}{\Lambda_{\text{sp}}} \right) \right]^{-1}$$  \(7\)

Approximate the external uncertainty of the exposure age $\sigma_{\text{ext} t}$, which includes the uncertainty in the production rate, in the same fashion. The uncertainties in the two reference production rates for simple and paleomagnetically-corrected age calculations happen to be the same, and we don’t take account of uncertainties in the paleomagnetic records. Thus, again for all practical purposes the external uncertainties are the same for the simple and paleomagnetically-corrected ages, and we therefore only calculate it once.

$$\left(\sigma_{\text{ext} t}\right)^2 = \left(\sigma_{\text{int} t}\right)^2 + \left(\sigma_{\text{ext} t}\right)^2$$  \(8\)

where

$$\frac{\partial t_{\text{simple}}}{\partial P_i} = -N_i \left[ P_i^2 - N_i P_i \left( \lambda_i + \frac{\rho E}{\Lambda_{\text{sp}}} \right) \right]^{-1}$$  \(9\)

and

$$\sigma_{P_i} = P_{i,\text{ref,mag}} \times S_{\text{thick}} \times S_T \times S_{i,\text{geo}}$$  \(10\)

The uncertainty estimation in this function could use some improvement. In future versions of this calculator, we should implement a Monte Carlo error-propagation scheme that can accommodate uncertainties in all the relevant parameters including the paleomagnetic records.

8. Calculates the paleomagnetically corrected exposure age $t_{\text{corrected}}$. To do this, it uses the internal MATLAB root-finding algorithm fzero to find the value of $t$ that zeros the objective function al_be_forward.m. As we describe later, the objective function integrates production over the duration of exposure $T$ to compute a normalized measured nuclide concentration at the present time:

$$\frac{N_i}{P_{i,\text{ref,mag}} S_{\text{thick}} S_T} = \int_{-T}^{0} S_{i,\text{geo}}(t) S_z(Et) e^{-\lambda_i t} dt$$  \(11\)
where, $P_{i,\text{ref,pmag}}$ is the reference production rate calculated using the paleomagnetically-corrected scaling scheme, $S_{i,\text{geo}}(t)$ describes the time-dependence of the geographic scaling factor as the geomagnetic latitude of the sample and the magnetic field strength vary, $S_z(z)$ describes the variation in production rate with depth, and $E$ is the erosion rate.

In the present function we compute the normalized nuclide concentration which forms the left side of the above equation. The objective function al_be_forward.m computes the integral on the right side of the equation for given $T$. We can then use the MATLAB function fzero to find the value of $T$ which solves the equation.

9. Finally, assemble the results, obtain diagnostic and version information from fzero and the objective function, and return a structure array containing the results.

The function is here:

http://hess.ess.washington.edu/math/AlBe/get_al_be_age.m

### 3.4 Objective function for paleomagnetic corrections: al_be_forward.m

This function takes five input parameters:

1. An exposure age $T$.
2. The structure array containing the sample information.
3. The structure array containing the constants.
4. A flag indicating the nuclide of interest, that is, 10 or 26.
5. A normalized nuclide concentration, that is, the measured nuclide concentration divided by the reference production rate, the thickness scaling factor, and the geometric shielding factor. Essentially this is the measured nuclide concentration with the time-invariant parts of the production rate divided out. It acts as the target against which the predicted nuclide concentration for the given exposure age $T$ will be evaluated.
6. A flag indicating whether or not to return diagnostic information.

The purpose of the function is to carry out a forward calculation of the nuclide concentration expected in the sample if it had been exposed for $T$ years, and return a measure of how well it fits the observed nuclide concentration in the sample. The value of $T$ that returns zero is then the actual exposure age of the sample. The forward calculation amounts to computing the predicted (normalized) concentration of nuclide $i$, by the following integral:

$$
\frac{N_i}{P_{i,\text{ref,pmag}}S_{\text{thick}}S_T} = \int_{-T}^{0} S_{i,\text{geo}}(t)S_z(z)S_T(E)e^{-\lambda_i t}dt
$$

(12)

where $t$ is taken to be in years before present. This takes into account paleomagnetic field variations by the dependence of $S_{i,\text{geo}}$ on $t$, the effect of radioactive decay by the exponential decay expression, and the effect of erosion by the additional scaling factor $S_z(z)$ which describes the dependence of the production rate on the depth $z$. The time dependence of the sample depth is simply $Et$, where $E$ is the erosion rate.

We actually do this integral as follows:
1. Create the time mesh. The meshpoints are spaced at 100-year intervals from 0-10,000 yr BP, and 500-year intervals thereafter.

2. Determine the time history of geomagnetic latitude $\phi(t)$ by computing the angular distance between the sample location and the magnetic pole position at each time step. The compilation of Pigati and Lifton (2004) gives magnetic pole positions every 100 yr for the past 10,000 yr. Assume that the geomagnetic pole coincides with the geographic pole for all time prior to 10,000 yr B.P.

3. Convert $\phi(t)$ to a cutoff rigidity vector $P_b(t)$ using the equation:

\[ P_b(t) = 14.9 \cos^4 \phi(t) M^*(t) \]  

where $M^*(t)$ is the time history of magnetic field strength relative to the present magnetic field, that is, the value of $M/M_0$ tabulated in Pigati and Lifton (2004) and their source references. First, this contains the so-called Elsasser equation relating cutoff rigidity to geomagnetic latitude for a simple dipole field. See, for example, Sandstrom (1965). Second, this takes account of past magnetic field variation by scaling the rigidity according to the ratio of past to present magnetic field strength. This is the same approximation inherent in the commonly used Nishiizumi (1989) scheme for paleomagnetic correction. We now know the cutoff rigidity at the sample site as a function of $t$ for the duration of exposure. The symbol $P_b$ for rigidity, which we inherit from Sandstrom, is unfortunate in light of all the other P’s that denote pressures and production rates.

4. Assemble a function $S_{i,sp}(P_b)$ that relates cutoff rigidity at the sample site to the scaling factor for spallogenic production. Here we use the scaling factors of Lal (1991) as modified by Stone (2000), with additional modifications to allow extension to cutoff rigidities not observed at present and thus application to low-latitude samples. Given the atmospheric pressure at the sample site $p$, the scaling factor for spallation as a function of atmospheric pressure is:

\[ S_{i,sp}(p) = a + b \exp \left( -\frac{p}{150} + cp + dp^2 + ep^3 \right) \]  

where $a...e$ depend on the cutoff rigidity and are defined for certain index rigidities:

<table>
<thead>
<tr>
<th>$P_b$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$M_{sl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.9</td>
<td>31.8518</td>
<td>250.3193</td>
<td>-0.083393</td>
<td>7.4260e-5</td>
<td>-2.2397e-8</td>
<td>0.587</td>
</tr>
<tr>
<td>14.02</td>
<td>34.3699</td>
<td>258.4759</td>
<td>-0.089807</td>
<td>7.9457e-5</td>
<td>-2.3697e-8</td>
<td>0.600</td>
</tr>
<tr>
<td>11.62</td>
<td>40.3153</td>
<td>308.9894</td>
<td>-0.106248</td>
<td>9.4508e-5</td>
<td>-2.8234e-8</td>
<td>0.678</td>
</tr>
<tr>
<td>8.38</td>
<td>42.0983</td>
<td>512.6857</td>
<td>-0.120551</td>
<td>1.1752e-5</td>
<td>-3.8809e-8</td>
<td>0.833</td>
</tr>
<tr>
<td>5.13</td>
<td>56.7733</td>
<td>649.1343</td>
<td>-0.160859</td>
<td>1.5463e-5</td>
<td>-5.0330e-8</td>
<td>0.933</td>
</tr>
<tr>
<td>2.54</td>
<td>69.0720</td>
<td>832.4566</td>
<td>-0.199252</td>
<td>1.9391e-5</td>
<td>-6.3653e-8</td>
<td>1.000</td>
</tr>
<tr>
<td>&lt; 0.93</td>
<td>71.8733</td>
<td>863.1927</td>
<td>-0.207069</td>
<td>2.0127e-5</td>
<td>-6.6043e-8</td>
<td>1.000</td>
</tr>
</tbody>
</table>

This table duplicates Table 1 in Stone (2000) except that latitude has been converted to cutoff rigidity via the Elsasser formula above in Equation 13.

The values in this table do not extend to high enough cutoff rigidities to allow computing $S_{i,sp}$ for samples at low latitudes during times of high magnetic field strength. We account for this by assuming that $\log S_{i,sp}$ is linearly related to $\log P_b$ for rigidities greater than 10 GV, following Quenby and Wenk (1962). We choose the slope of the relationship by fitting to the values for all the index rigidities above 10 GV. We extrapolate to 21 GV, which is the approximate cutoff rigidity at the equator during times of highest past magnetic field strength. Thus, to assemble the function that gives $S_{i,sp}$ as a function of cutoff rigidity, we calculate $S_{i,sp}$ for all the index rigidities at the atmospheric pressure at the sample site, then extrapolate to 21 GV as discussed above. This results in a function $S_{i,sp}(P_b)$ that is defined at certain values of $P_b$.

5. Assemble a similar function $S_{i,\mu}(P_b)$ that relates cutoff rigidity to the scaling factor for production by muons. Again we follow Lal (1991) and Stone (2000) and define the scaling factor for production by muons as a function of atmospheric pressure by:

\[ S_{i,\mu}(p) = a + b \exp \left( -\frac{p}{150} + cp + dp^2 + ep^3 \right) \]
where the muon scaling factor at sea level $M_{sl}$ is again defined at certain index rigidities in the table above. We extend the defined constants to higher rigidities by the same assumption that $\log M_{sl}$ is linearly related to $\log P_b$ at rigidities above 10 GV.

6. Use the two functions relating rigidity to scaling factors for production by spallation and by muons to convert the time history of cutoff rigidity at the sample site $P_b(t)$ to time-dependent scaling factors $S_{i,sp}(t)$ and $S_{i,\mu}(t)$. We use linear interpolation between the defined values of $S_{i,sp}(P_b)$ and $S_{i,\mu}(P_b)$ to avoid the well-known problem of under- and over-shooting between index latitudes due to the polynomial form of the Lal equations.

7. Combine the two scaling factors as follows to obtain the time-dependent normalized production rate history for the sample, as required in Equation 12:

$$S_{i,geo}(t) = f_{sp} \left( S_{i,sp} e^{\frac{E_t}{\lambda_{sp}}} \right) + (1 - f_{sp}) \left( S_{i,\mu} e^{\frac{E_t}{\lambda_{\mu}}} \right)$$

(16)

Where the exponential terms approximately account for the effect of erosion on changing the production rate in the sample itself over time. The factor $f_{sp}$ is taken from Stone (2000). The use of a single attenuation length for muons is highly approximated.

8. Integrate the production rate history to get the predicted nuclide concentration for the given $t$, as follows:

$$t_{predicted} = \int_{-t}^{0} S_{i,geo}(t) S_z(E_t) e^{-\lambda_{i}t} dt$$

(17)

We do this by trapezoidal integration using the internal MATLAB function trapz.

9. Finally, difference the predicted nuclide concentration and the normalized measured nuclide concentration provided as a target to determine the misfit between the two. Return the misfit.

### 3.5 Subsidiary functions

#### 3.5.1 Angular distance measurement: angdist.m

This function calculates the angular distance between two points on a sphere. Given (latitude, longitude) for two points $(\phi_1, \theta_1)$ and $(\phi_2, \theta_2)$, the angle between them is:

$$arccos \left( \cos(\phi_1) \cos(\phi_2) \left[ \cos(\theta_1) \cos(\theta_2) + \sin(\theta_1) \sin(\theta_2) \right] + \sin(\phi_1) \sin(\phi_2) \right)$$

(18)

#### 3.5.2 Elevation/pressure relationships: stdatm.m and antatm.m

These functions calculate the atmospheric pressure at given elevation for the ICAO standard atmosphere and an approximation for the Antarctic atmosphere. For details, see Stone (2000).
3.5.3 **Angle conversions: r2d.m and d2r.m**

These functions convert radians to degrees, and vice versa.

3.5.4 **Geographic scaling factor: stone2000.m**

This function determines the production rate scaling factor for a particular atmospheric pressure and latitude following the procedure in Stone (2000).

3.5.5 **Thickness scaling factor: thickness.m**

This function computes a scaling factor that takes account of the thickness of the sample by integrating the production rate-depth relationship over the sample thickness. It assumes that the production rate decreases with depth according to a single exponential function:

\[
P(z) = P(0) \exp\left(\frac{\rho z}{\Lambda_{sp}}\right)
\]  

where \(P(0)\) is the surface production rate and \(z\) is depth below the surface (cm). The thickness scaling factor \(S_{\text{thick}}\) is then:

\[
S_{\text{thick}} = \frac{\Lambda_{sp}}{\rho z} \left[ 1 - \exp\left( -\frac{\rho z}{\Lambda_{sp}} \right) \right]
\]  

3.5.6 **Geometric shielding: skyline.m**

[This needs some content. For now, there is a description of the function in the MATLAB code itself.]

4 **References**

Gosse and Phillips  
Lal 1991  
Middleton and Klein 1987  
Nishiizumi 1989  
Pigati and Lifton 2004
Sandstrom 1965

Stone 2000

Citations for $^{10}\text{Be}$ and $^{26}\text{Al}$ calibration measurements